Investigation of the asymptotic states of granular materials using a discrete model of anisotropic particles

A.A. Peña & H. J. Herrmann
Institute of Computer Physics, University of Stuttgart, Germany

A. Lizcano
Departamento de Ingeniería Civil, Universidad de los Andes, Colombia

F. Alonso-Marroquin
Department of Mechanics, National Technical University of Athens, Greece

ABSTRACT: We study the mechanical response of non-cohesive granular materials under monotonic and cyclic loading using a discrete model of polygonal particles. Three asymptotic states are studied: The first is the so-called critical state in which dense and loose media achieve the same void ratio at large strains independently of their initial density state. The second is related to the loss of memory when the granular media is compressed using constant strain rates. This proportional compression is connected with proportional stress paths independent of the initial stress state. The third state is observed under cyclic loading. This is given by constant accumulation of permanent deformation that is called ratcheting. These asymptotic states are studied at the micro-mechanical level by following the evolution of the fabric tensor of the contact network.

1INTRODUCTION

Granular materials exhibit a complex mechanical behavior when they are loaded. This macro-mechanical response is a result of the discrete character of the media, and depends on the grains themselves (shape, angularity, size and so on), the evolution of the granular skeleton (void ratio, fabric), and some phenomena occurring at the grain scale (rolling, sliding). This global response also involves the existence of the so-called asymptotic stress-strain behaviors, which are a very important feature in granular soils. These asymptotic states are independent of the initial state of the material, and they have been used as fundamental principles for the formulation of several constitutive relations (Schofield & Wroth 1968, Gudehus 1997). By means of numerical simulations, it has been possible to study different phenomena occurring at the micro-mechanic scale (Cundall et al. 1979; Alonso-Marroquín & Herrmann 2004), and to reproduce the global response observed in realistic experiments of granular materials (Peña et al. 2004).

In this work, we use the molecular dynamic technique to simulate mechanical behavior of two-dimensional particle assemblies contained by wall boundaries. The grains are represented by randomly generated convex polygons. This method captures the polydispersity of the sizes, as well as the angularity and the inherent anisotropy of the grains.

We present results of monotonic and cyclic shear tests, and compression with constant strain rates in order to study the influence of particle anisotropy (shape or elongation) on the asymptotical behaviors of granular materials: the so-called critical state, the loss of memory, and granular ratcheting. Results are analyzed from the macro and micro-mechanical point of view.

2BIAXIAL TESTS

Initially, isotropic polygons are generated by means of a Voronoi tessellation based on a square lattice. In order to generate elongated particles, the isotropic polygons are dilated (contracted) in the horizontal (vertical) direction by a given factor, respectively. The ratio between these factors gives the average elongation ratio of the grains. We use three different elongation ratios: 1.0, 1.8 and 2.3.

The specimens are constructed by starting from very loose samples, which are compressed isotropically until the desired confining pressure is reached. Elongated particles, however, present at the end of the process a structural anisotropy due to their trend to take preferential orientations during the compression. Dense samples are created by setting the interparticle friction to zero, and the loose ones are obtained taking damping coefficients 100 times greater than those used in the loading stage. In all the simulations the mechanical properties of the particles were: interparticle friction coefficient μ = 0.55, normal stiffness k_n at the contact = 160 Mpa, and normal damping coefficient = 4000 s^{-1}. We used 400 polygons.

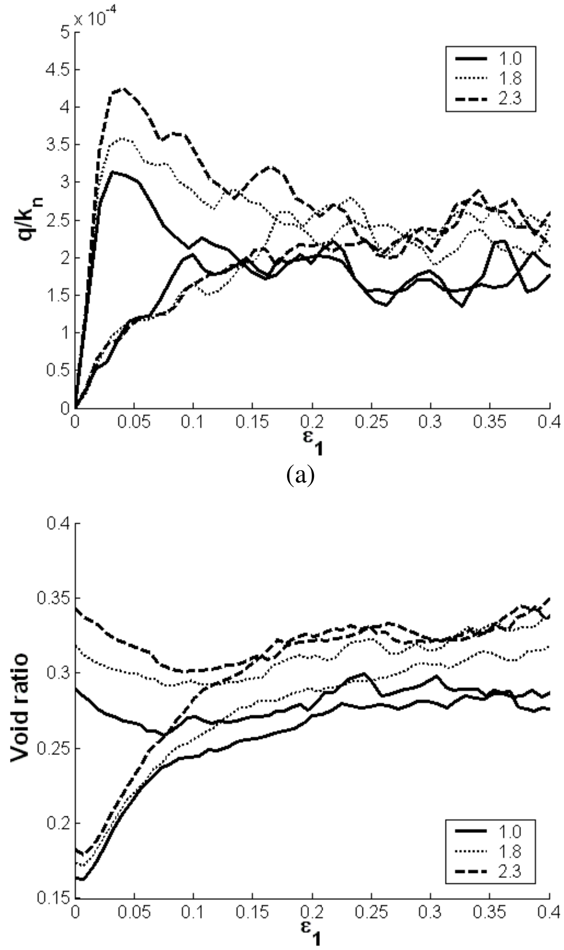
The rigid walls confining the sample are frictionless, and therefore they only transmit normal forces. These boundary forces are applied on each grain in contact with these walls. The displacement of the walls and the total force acting on them are used to determine the global strain and stress of the assembly. The axial and lateral directions are indicated as 1 and 2, respectively. Stresses have the same sign convention used in soil mechanics: compressive normal stresses are positive. The stress values are normalized by the normal stiffness at the contacts k_n.

The anisotropy of the contact network is characterized by the fabric tensor **F**, whose components are obtained from the mean value $F_{ij} = \langle n_i \, n_j \rangle$, where n_i is the normal unit vector at a contact point. The trace of this tensor corresponds to the mean coordination number of the polygons, and the deviatoric part (F_1-F_2) is used to study the evolution of the structural anisotropy within the granular media.

2.1 *Monotonic shearing – Critical State*

In this test, the sample is loaded using strain control on the horizontal walls (the axial strain rate is set to 0.02 s^{-1}), while the stress on the vertical walls remains constant at 16 kPa. The results of the test for the different elongation ratios are presented in Figure 1. As expected, in Figure 1(a) dense samples exhibit a higher initial stiffness and a peak value, after which strain-softening behavior is observed. The loose media do not exhibit a peak. Samples with a large elongation ratio reach higher deviatoric stress, but at the same time they exhibit higher stress fluctuations around a value that one could consider as a steady state of the material.

In Figure 1(b) the evolution of the void ratio with axial strain is illustrated. Initially, dense samples contract and later expand. For large axial strain values, both dense and loose samples with the same elongation ratio reach a constant void ratio. This corresponds to the so-called critical state of the material and it is independent of the initial density state (Schofield and Wroth 1968). It is evident that, as expected, samples with different elongation ratio reach different void ratios at the critical state. The bigger the anisotropy, the bigger the void ratio and axial strain to reach the critical state. Therefore, granular media with elongated particles are more sensitive to volumetric changes or dilatancy due to the bigger interlocking among particles, and consequently a bigger shear strength is also developed. This states the important role of shape and angularity of particles on the mechanical behavior of the granular media.



(a)

(b)

Fig. 1. Evolution of (a) the deviator stress (b) void ratio during the biaxial test for different ratios of particle elongation.

The elongated particles, the initial anisotropy is partially erased evolution of the induced structural anisotropy during the biaxial test is shown in Figure 2. Initially, the deviatoric fabric increases for all the samples in the direction of loading independent on the initial anisotropy. In case of and re-oriented. In general, dense samples present a bigger increment of anisotropy, but at around 15% of axial strain dense and loose samples (independent of initial density state and structural anisotropy) reach the same value of deviatoric fabric within statistical error. This result coincides with the asymptotical features: critical void ratio and steady state during monotonic shearing, and implies the existence of a critical anisotropy in which the average orientation of all the contacts is independent on the initial density. It coincides with results obtained by Nouguier et al. (2003).

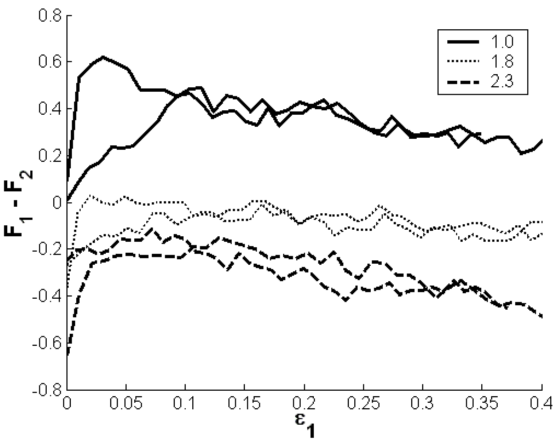
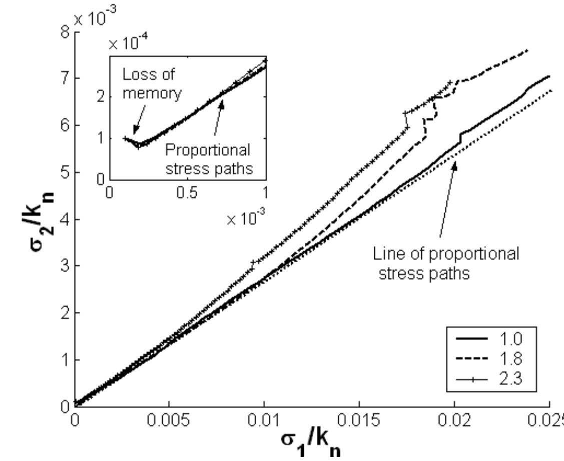


Figure 2. Evolution of anisotropy during the biaxial test for different ratios of particle elongation.

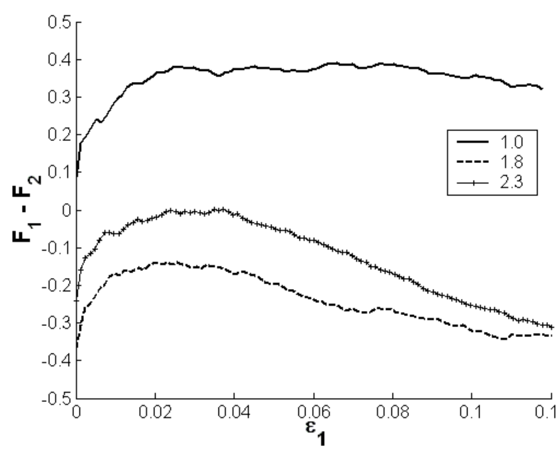
2.2 *Proportional compression – Loss of memory*

Soils are able to memorize previous loading, but under certain type of deformation, samples reach a state in which they loss their memory. This loss of memory has been experimentally observed by Goldscheider (1976) and others. This behavior is the so-called Swept out of Memory (SOM), Gudehus et al. 1977. This loss of memory occurred when samples approaches asymptotically to proportional stress paths, inset of Figure 3(a), under uniform shortening with constant strain rates (proportional compression).

In Figure 3 results of the proportional compression test (rate $\varepsilon_2/\varepsilon_1 = 0.33$) on isotropic and anisotropic particles are presented. We observe in inset of Figure 3(a) that at the beginning samples reach proportional stress paths, i.e. stress paths tend to straight lines with a slope σ_2/σ_1. This range of stress corresponds to circa 4% of axial strain. In Figure 3(b) we notice that independent of the initial structural anisotropy (deviatoric fabric), all the samples' anisotropy evolves towards the main direction of loading (axial), and reaches a plateau which remains approximately constant. Samples consisting of elongated particles tend to recover their initial value of anisotropy for large axial strain. This is in contrast to the case of isotropic particles, where the sample seems to remain close to a constant deviatoric fabric and lose its memory. This evolution of the structural anisotropy is related to the stress evolution of the samples. Isotropic polygons follow the proportional stress paths of the SOM behavior, but elongated polygons deviate from this asymptotic state. At large strains, isotropic polygons also deviate from the SOM. This coincides with a slightly decrease of deviatoric fabric. Based on these results, we concluded that the existence of the memory-free behavior could require the existence of a constant value of structural anisotropy, but different from the initial one. It is important to point out that the initial structural anisotropy is different for isotropic and anisotropic polygons. Therefore, the influence of particle shape and initial structural anisotropy on the mechanical behavior, requires a deeper investigation.



(a)



(b)

Figure 3. Stress paths for proportional compression (a) the inset shows the initial memory-free behavior, and evolution of deviatoric fabric (b) obtained in numerical simulation

2.3 *Cyclic shearing – granular ratcheting*

The so-called *granular ratcheting* results from the asymptotic response of samples subjected to moderately small load amplitudes (Alonso-Marroquín & Herrmann 2004). Granular ratcheting is nothing more than a limit behavior of the cyclic loading where the stress-strain curve is given by the same hysteresis loop, Figure 4(a). This loop produces not only a constant power dissipation per cycle (Garcia-Rojo et al. 2004), but also a constant accumulation of permanent shear deformation $\Delta\gamma$. The latter is shown in Figure 4(b). Samples with different inherent anisotropy all reach the ratcheting regime, and present a different Δγ. However, the dependence of this constant accumulation of permanent strain on inherent anisotropy of the grains does not exhibit a systematic trend, and is not therefore clear at the moment.

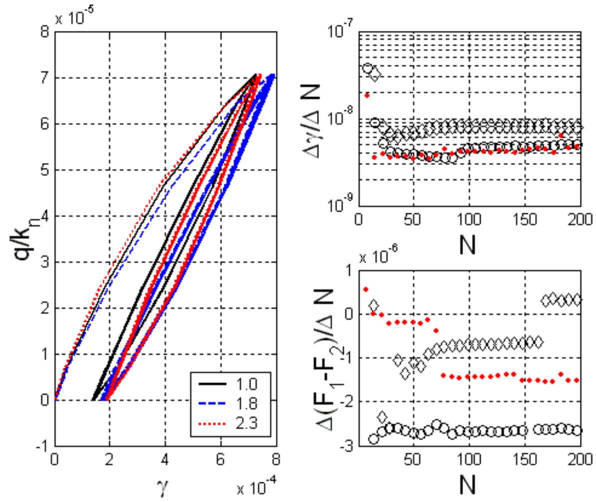


Figure 4. (a) Hysteresis loops of the granular ratcheting (b) Permanent accumulation of plastic deformation per cycle for shape ratios of 1.0 (circles) 1.8 (diamonds) and 2.3 (points) (c) Variation of the deviatoric part of the fabric tensor for the three shape ratios given in (b).

The fabric tensor provides some interesting micro-mechanical features of the granular ratcheting. The trace of this tensor ($F_1 + F_2$) stays almost constant in all the simulations. This is due to the fact that opening and closure of contacts are quite rare events during the cyclic loading. A significant number of contacts reaches almost periodically the sliding condition. They lead to a systematic shift of the grains against each other, which turns out in a slow convection movement within the packing (Alonso-Marroquín 2004). This rearrangement of the granular skeleton is reflected in a constant variation of anisotropy with the number of cycles, as shown in part (c) of Figure 4. Note that the rate of F_1 - F_2 is not always constant, but can change abruptly and may even change of sign. Actually, the accumulation of permanent deformation at the contacts with the number of cycles is not strictly linear, but there are short time regimes during which granular skeleton rearranges itself, driven to a new ratcheting regime (Alonso-Marroquín & Herrmann 2004).

3CONCLUSIONS

In this paper, using discrete numerical simulations, preliminary results concerning three asymptotic behaviors of granular media are presented. The importance of particle shape in the mechanical behavior of granular material is addressed. Performing monotonic shear tests, samples reach the so-called critical state, independent on their initial density and particle shape, and they deform at a constant deviator stress, volumetric strain and structural anisotropy. During the proportional compression test, the proportional stress paths of the memory-free behavior are initially reached for all the samples, but at larger axial strains, elongated particles deviate from this state. It seems that structural anisotropy may affect this limit behavior, but further numerical simulations should be performed to confirm this hypothesis. Samples under cycling shearing all reach the ratcheting behavior. This is given by a periodic rearrangement of the contact network involving long time regimes with constant variation of anisotropy. Do samples with different void ratios reach the same ratcheting behavior? What is the relation between the limit hysteresis loops and microstructure?. The answer to these questions will require more expensive calculations.

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REFERENCES

Alonso-Marroquín, F. & Herrmann, H.J. 2004. Ratcheting of granular materials. *Phy. Rev. Lett.* 92, 054301.

Alonso-Marroquín, F. 2004. Incremental response and granular ratcheting. *PhD thesis*, University of Stuttgart, Germany, Logos Verlag Berlin ISBN 3-8325-0560-1.

Cundall, P.A. & Strack, O.D.L. 1979. A discrete numerical model for granular assemblies, *Géotechnique* 29, No. 1, 47-65.

Garcia-Rojo, R. & Herrmann, H.J. 2004. Shakedown of unbound granular materials. Granular Mater. cond-mat 0404176, accepted for publication.

Goldscheider, M. 1976. Grenzbedingung und Fliessregel von Sand. Mechanics Research Communications 3: 463 – 468.

Gudehus, G. 1977. Stability of saturated granular bodies under cyclic load. *In: Plastic and long-term effects in soils* (Editor G.Gudehus), Balkema, DMSR, Karlsruhe.

Gudehus, G. 1997. Attractors, percolation thresholds and phase limits of granular soils, *Proceedings Powders & Grains 97,* Behringer and Jenkins, 169-183.

Nouguier, C. Cambou, B. & Vincens, E., 2003. Influence of particle shape and angularity on the behavior of granular materials: a numerical analysis, *Int. J. Numer. Anal. Meth. Geomech.,* **27**: 1207-1126.

Peña, A.A. Lizcano, A, Alonso-Marroquín, F. & Herrmann H.J. 2004. Numerical simulations of biaxial tests using non-spherical particles. Submitted to *Int. J. Numer. Anal. Geomech.*

Schofield, A.N. & C.P. Wroth. 1968. *Critical State Soil Mechanics*, McGraw Hill, Maidenhead.