

Simulation of ice-sheet melting

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Abstract.

A simple model is presented where the temperature, extent and thickness of sea ice is predicted in a numerical simulation. The model is based on Newton's law of cooling with added terms to account for black-body emission by the ice and solar and atmospheric radiation. The ice is assumed to be an ellipsoid, with the temperature being uniform throughout. Coupled equations are developed for the temperature of the ice and the water beneath it. A second model is presented based on the one-dimensional heat equation, where the ice is considered as a uniform, infinitely large slab of ice. The added terms in the first model are used as boundary conditions and the ice initially set to have a uniform temperature throughout. The first model predicts that the ice sheet is completely melted in less than a month. This is largely due to the initial assumption that the temperature of the ellipsoidal ice is uniform. The second model predicts a melting time in the order of a month, depending on the number of layers of ice used. The fast melting time is a consequence of assuming that the temperature of the water beneath the ice sheet remains constant during the melting and heating processes. While the models are simple and reasonably fast computationally, more realistic time-scales can be obtained through revision of the initial assumptions and the development of a more complicated model.

Keywords: sea ice; modelling; numerical analysis; melting

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1. INTRODUCTION

With increasing attention being paid to global warming, especially in the Arctic and Antarctic regions, the global climate system is becoming an important area of research. Predicting changes in climate is a complex task due to the many relationships involved between components, such as the ocean, atmosphere and land. Sea ice is also an important element to include. It affects the heat transfer between ocean and atmosphere and warms the atmosphere due to its high reflectivity (albedo).

The thickness of sea ice is much smaller than its extent. Changes in climate can therefore have a major impact on the extent of the ice. This effect is only reinforced by the positive ice-albedo feedback system. As the ice melts and decreases in thickness, the albedo of the ice also decreases. The lower albedo in turn causes an increased rate in melting, further reducing the thickness of the ice.

In recent years, the extent and thickness of Arctic sea ice have reached record lows. The United States National Oceanic and Atmospheric Administration reported that the summer Arctic sea ice extent during 2008 was only 4.7 million km², while in winter months this expanded to 15.2 million km². This is markedly lower than the average summer and winter extents during 1979-2008 of 6.7 and 15.6 million km². The annual mean thickness has seen a reduction of 34% from its peak value of 3.71 m in 1980 to 2.46 m in 2007 [1].

A pioneering model was developed by Maykut and

Untersteiner (1971) for Arctic sea ice [2]. Their one-dimensional model incorporated these and other thermodynamic factors and predicted the temperature profile, thickness and surface melt of Arctic ice. This was later simplified to a three-layer and even simpler zero-layer model by Semtner (1976) [3]. More recently, Bitz and Lipscomb (1999) developed an energy-conserving model, taking into consideration the dependence of heat capacity on temperature as well as internal storage of energy in pockets of brine within the ice [4].

While these models are particularly detailed, it is often desirable to use a simple model for use in a larger global climate system. In this paper, a simple, time-dependent heat flux model is developed, involving heat transfer between the ice, ocean and atmosphere. The model predicts the temperature, mass, area and thickness of the ice and surrounding ocean during the melting and heating processes, assuming the temperature of the ice is uniform throughout. A model based on the one-dimensional heat equation is also briefly investigated, in which the temperature profile of the ice is predicted.

2. THE MODEL

The heat flux model, shown in Fig. 1, assumes that the ice is an ellipsoid and that the temperature of the ice is uniform throughout. The model is based on Newton's law of cooling - the change in temperature of the ice dT_i/dt is proportional to the temperature difference be-

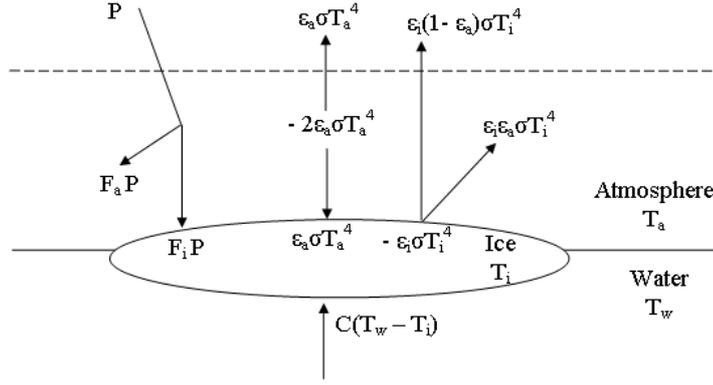


FIGURE 1. A diagram of the heat flux model, based on Newton's law of cooling (adapted from [5]).

tween the ice and water beneath it, $h(T_w - T_i)$. Additional terms are added to account for an increase in temperature due to radiation from the Sun $F_i P$ and atmosphere $\epsilon_a \sigma T_a^4$, and a decrease in temperature due to black-body emission by the ice, $-\epsilon_i \sigma T_i^4$. The melting process of the ice is represented by the dm_i/dt term and starts when the temperature of the ice reaches the melting temperature -1.8°C [2]. Melting is assumed to be isothermal, that is, $dT_i/dt = 0$; similarly, during the heating process $dm_i/dt = 0$ and no mass is lost.

These terms form the main equation describing the heating and melting process of the ice

$$c_{v,i} m_i \frac{dT_i}{dt} = A_i [C(T_w - T_i) + F_i P + \epsilon_a \sigma T_a^4 - \epsilon_i \sigma T_i^4] + \lambda \frac{dm_i}{dt}. \quad (1)$$

While heating, the temperature of the ice is a function of the time t (in seconds) and the water and atmosphere temperature T_w and T_a . The mass m_i and area A_i of the ice are constant during this time. The area A_i is the surface area of the ice that is in contact with the water or atmosphere. It is assumed to be half the total surface area of an ellipsoid; that is, half of the ice is in contact with the water beneath and half with the atmosphere above.

The constants are the constant-volume heat capacity of ice $c_{v,i} = 2050 \text{ J kg}^{-1} \text{ K}^{-1}$, heat transfer coefficient for water $h = 500 \text{ J s}^{-1} \text{ K}^{-1} \text{ m}^{-2}$ [6], fraction of incident solar radiation absorbed by the ice $F_i = 0.5$ [7], incident solar radiation $P = 1365/4 \text{ J s}^{-1} \text{ m}^{-2}$ [5], emissivity of ice and water, $\epsilon_a = 0.9$ [5] and $\epsilon_i = 0.97$ [8] respectively, and latent heat of fusion for ice $\lambda = 333 \times 10^{-3} \text{ J kg}^{-1}$. The fraction of absorbed radiation has been defined so that $F = 1 - \alpha$, where α is the albedo (reflectivity) of the surface. The Stefan-Boltzmann constants is $\sigma = 5.67 \times 10^{-8} \text{ J s}^{-1} \text{ m}^{-2} \text{ K}^{-4}$.

Eq. (1) is coupled with a similar equation for the water beneath the ice

$$c_{v,w} m_w \frac{dT_w}{dt} = A_w [C(T_i - T_w) + F_w P + \epsilon_a \sigma T_a^4 - \epsilon_w \sigma T_w^4] + \lambda \frac{dm_w}{dt}. \quad (2)$$

Here $c_{v,w} = 4181 \text{ J kg}^{-1} \text{ K}^{-1}$, $F_w = 0.94$ [9] and $\epsilon_w = 0.44$ [8].

The temperature of the atmosphere T_a used in Eq. (1) and (2) is obtained from balancing fluxes at the top of the atmosphere. This yields

$$(F_a + F_i)P - \epsilon_a \sigma T_a^4 - (1 - \epsilon_a) \epsilon_i \sigma T_i^4 = 0. \quad (3)$$

During melting, it is assumed that the process is isothermal and that the ice melts such that it remains an ellipsoid, with half of its surface area always in contact with the water and half with the atmosphere. The mass loss rate of the ice can be found from Eq. (1), setting $dT_i/dt = 0$. To determine the time-dependent area A_i and thickness z of the ice, assume that

$$A_i(t) = A_{0i} r_i^2(t) \quad (4)$$

$$z(t) = z_0 r_i(t) \quad (5)$$

where A_{0i} and z_0 are the area and thickness of the ice before melting and $r_i(t)$ is some time-dependent function yet to be determined.

Since the thickness of the ice is much smaller than its areal extent (section 1), approximate the total area of an ellipsoid to be $2\pi ab$. Given that the volume of an ellipsoid of xyz -dimensions a , b and c is $\frac{4}{3}\pi abc$, the volume of the ice expressed in terms of its surface area and thickness is $2/3 A_i z$. The mass of the ice is therefore

$$m_i = \rho_i \frac{2}{3} A_i z = \rho_i \frac{2}{3} A_{0i} z_0 r_i^3$$

where $\rho_i = 900 \text{ kg m}^{-3}$ is the density of the ice and Eq. (4) and (5) have been used for A_i and z respectively. The above equation can now be solved for the time-dependent function $r_i(t)$

$$r_i(t) = \sqrt[3]{\frac{m_i(t)}{\rho_i \frac{2}{3} A_{0i} z_0}}. \quad (6)$$

Conservation of mass implies that the mass of the water must satisfy

$$\frac{dm_i}{dt} = \frac{dm_w}{dt}. \quad (7)$$

Since the area of the ice and water remains constant during melting, the area of the water is determined simply by using

$$A_w(t) = A_{0i} + A_{0w} - A_i(t) \quad (8)$$

where A_{0w} is the area of the water before melting.

Differential equations (1) and (2) are numerically solved for the ice and water temperatures T_i and T_w during heating using the fourth-order Runge-Kutta method and Eq. (3), setting $dm_i/dt = dm_w/dt = 0$. When the temperature of the ice is above the melting temperature 271.2 K (-1.8°C), T_i must be set to 271.2 K. During melting, Eq. (1) and (7) are solved for ice and water masses m_i and m_w in a similar manner, setting $dT_i/dt = 0$. Once m_i is known, and using Eq. (6), Eq. (4), (5) and (8) are easily solved for the ice area A_i , thickness z and water area A_w .

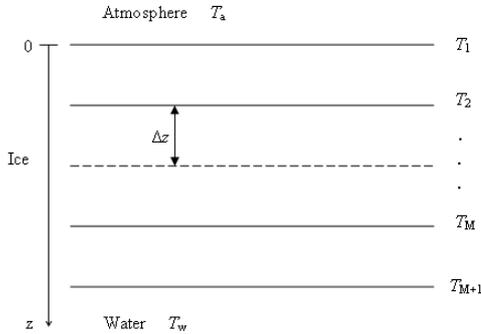


FIGURE 2. A diagram of the one-dimensional heat equation model with M number of layers.

A model based on the heat equation, shown in Fig. 2 was also briefly investigated as a follow-up the heat flux model. Unlike the heat flux model, it does not assume that the temperature of the ice is uniform throughout. Since the extent of the ice is much larger than the thickness, the ice is treated as a flat, infinite long slab having thickness z_0 . The partial differential equation is then reduced to one dimension, the z -direction

$$\frac{\partial T}{\partial t} = \frac{k_i}{\rho_i c_{p,i}} \frac{\partial^2 T}{\partial z^2}. \quad (9)$$

This corresponds to the thickness of the ice sheet, with the atmosphere/ice interface at $z = 0$ and the ice/water interface at $z = z_0$. Note that the temperature T refers to the temperature of the ice unless otherwise specified; the subscript has been dropped to avoid confusion in later equations. The constant $k_i = 2.215 \text{ J s}^{-1} \text{ m}^{-1} \text{ K}^{-1}$ [10] is the thermal conductivity of the ice. The constant-pressure heat capacity $c_{p,i}$ is assumed to be equal to the constant-volume heat capacity $c_{v,i}$ mentioned previously. For simplicity, it is assumed that the heat energy released into the atmosphere and water by the ice does not significantly change their temperatures; hence the atmospheric and water temperatures remain constant. The atmospheric temperature is fixed at $T_a = -20^\circ\text{C}$ (2543 K) and the water temperature at $T_w = 0^\circ\text{C}$ (273 K), slightly above the melting point.

The initial conditions for Eq. (9) are set to be at some constant temperature, above the melting point

$$T(0, z) = 269 \text{ K} \quad (-4^\circ\text{C}) \quad (10)$$

The boundary conditions are taken from the terms in Eq. (1). At the upper boundary ($z = 0$), the ice gains heat energy due to solar radiation and radiation from the atmosphere, and loses heat due to black body emission

$$\left. \frac{\partial T}{\partial z} \right|_{z=0} = \frac{1}{k_i} (F_i P + \epsilon_a \sigma T_a^4 - \epsilon_i \sigma T^4). \quad (11)$$

At the lower boundary ($z = z_0$), the ice gains heat energy due to the warmer ocean beneath it

$$\left. \frac{\partial T_i}{\partial z} \right|_{z=z_0} = -\frac{C}{k_i} (T - T_w) \quad (12)$$

Eq. (9) – (12) are numerically solved using the implicit backward Euler method. The z -dimension of the ice sheet is divided into M layers, resulting in $M - 1$ interior layers and $M + 1$ temperature points. The first order derivative in Eq. (9) is approximated by applying the backward-difference formula, and the second derivative by applying the centred-difference formula

$$\frac{\partial^2 T}{\partial z^2} \approx \frac{1}{\Delta z^2} (T_{i+1,j} - 2T_{i,j} + T_{i-1,j})$$

$$\frac{\partial T}{\partial t} \approx \frac{1}{\Delta t} (T_{i,j+1} - T_{i,j})$$

where Δt is the time step in seconds and Δz is the space step in metres. Subscripts $i = 1 \dots M + 1$ and $j = 1 \dots N$ refer to the layer and time step number respectively, where M is the number of layers of ice and N is the number of time steps. They satisfy the conditions $z = (i - 1)\Delta z$ and $t = (j - 1)\Delta t$. Using these formulae in Eq. (9) and solving for the temperature at the next time step $T_{i,j+1}$ yields

$$T_{i,j+1} = sT_{i+1,j} + (1 - 2s)T_{i,j} + sT_{i-1,j} \quad (13)$$

where $s = \frac{k_i}{\rho h_0 c_p i} \frac{\Delta t}{\Delta z^2}$.

The boundary conditions, corresponding to $i = 1, M + 1$, are discretized in a similar manner using backward-differences, resulting in

$$T_{1,j+1} = \frac{T_{i,j+1} + \Delta z T_w \frac{C}{k_i}}{1 + \Delta z \frac{C}{k_i}} \quad (14)$$

$$T_{M+1,j+1} = \frac{\Delta z}{k_i} (F_i P + \epsilon_a \sigma T_a^4 - \epsilon_i \sigma T_{M+1,j+1}^4) + T_{M,j+1} \quad (15)$$

Eq. (13) is used in matrix form ([11]) to calculate the temperatures of the interior layers of ice ($i = 2, \dots, M$) using temperatures from the previous time step. The temperatures of the boundaries at $i = 1, M + 1$ can then be calculated using Eq. (14) and (15), solving Eq. (15) as a fourth-degree polynomial.

Melting starts when the bottom layer of ice ($i = M + 1$) heats to the melting temperature T_m or above (the model assumes that the upper boundary temperature is always less than the melting point). The layer is assumed to melt instantaneously; its individual mass Δm is set to zero and its temperature is set equal to T_m . The mass and height of the entire ice sheet are decreased by Δm and Δz respectively. The one-dimensional heat equation is re-applied to the shortened ice sheet, with $M = M - 1$. The boundary conditions remain the same and the initial conditions are taken from the temperature profile from the previous time step. If there are layers below the bottom layer of ice that are already melted, their temperatures are set to the temperature of the water T_w . This is a originates from the previous assumption that the temperature of the water beneath the ice does not change significantly during the melting and heating processes.

3. RESULTS

In both models, we use as input values for the Arctic ice and surrounding ocean. The initial area of the Arctic ice is $A_{0i} = 14 \times 10^{12} \text{ m}^2$ with thickness $z_0 = 2.5 \text{ m}$ and mass $m_{0i} = 2.1 \times 10^{16} \text{ kg}$. The initial area of the ocean is $A_{0w} = 14 \times 10^{12} \text{ m}^2$ with mass $m_{0w} = 1.5 \times 10^{19} \text{ kg}$ [1]. In the heat flux model, the initial temperature of the ice is below the melting temperature T_m at $T_{0i} = 269 \text{ K}$ (4°C) and the initial temperature of the water is slightly above T_m at $T_{0w} = 273 \text{ K}$ (0°C). The was run for a period of $2 \times 10^6 \text{ s}$ with a 100-second time step. The heat equation model used $M = 250$ layers, corresponding to layers of thickness $\Delta z = 1 \text{ cm}$. It was run for a period of $5 \times 10^6 \text{ s}$ with $N = 50001$, corresponding to a 100-second time step. The results for the heat flux model are shown in Fig. 3. The time scale is logarithmic in order for both

the heating and melting processes to be viewed. The top row of graphs correspond to the ice and the bottom row to the water. The ice heats from the initial temperature -4°C to the melting temperature -1.8°C in 92 min (5,500 s), corresponding to an average heating rate of 1.4°C/hr . During the melting process, the temperature of the ice and water remain constant due to the assumption that the process is isothermal. The ice melts completely in 0.65 mths ($1.7 \times 10^6 \text{ s}$), implying that the area and height of the ice decrease at an average rate of 22 million km^2/mth and 3.8 m/mth respectively.

Table 1 shows the effect of each flux term in Eq. (1), and of varying the F_i , P and C parameters. With the exception of case 5, the time taken for the ice sheet to heat and melt completely remains between 0.5 and 1 month. Cases 1 and 6 indicate that although including radiation and emission terms decrease the melting time, they do not significantly affect the order of the time-scale, with melting occurring in less than a month in both cases. From cases 2 to 5, the most important term in the model is the heat flux from the ocean, followed by solar radiation. The least important terms are atmospheric radiation and energy emission from the ice.

Now we look at the effect of varying parameters in Eq. (1), cases 7 to 12. The model is most sensitive to variations in the heat transfer coefficient C . Lowering it to a value of $400 \text{ J s}^{-1} \text{ m}^{-2} \text{ K}^{-1}$ causes the melting time to increase by 22%, while raising it to $700 \text{ J s}^{-1} \text{ m}^{-2} \text{ K}^{-1}$ results in a decrease of 27%. The incident solar radiation and fraction of it absorbed by the ice are less important in the model, causing differences of up to 14% in melting time.

If the atmospheric and water temperatures are fixed at 253 K and 273 K respectively, the melting time does not change notably. This means that these temperatures do not vary significantly in the model.

Next we look at the results of the heat equation model. If all terms in Eq. (11) are used, the surface temperature of the ice ($i = 1$) rises above the melting point. This violates the original assumption that no melting occurs at the upper boundary. In keeping with this assumption, the solar radiation term $F_i P$ is removed from Eq. (11).

Fig. 4 shows selected temperature profiles with the modified boundary condition. The time-step size was increased to 10,000s instead of the 100s step-size used in the previous model to decrease computation time. There are 250 layers of ice (1 cm thick). The bottom layer of ice heats to the melting temperature instantaneously, resulting in the ice sheet heating and melting simultaneously. As multiple layers of ice are melted, their temperatures become equal to the temperature of the water $T_w = 273 \text{ K}$. This corresponds to the flat sections of the temperature profile at 273 K. The temperature through the ice reaches a minimum at the atmosphere/ice boundary due to the low atmospheric temperature and black-

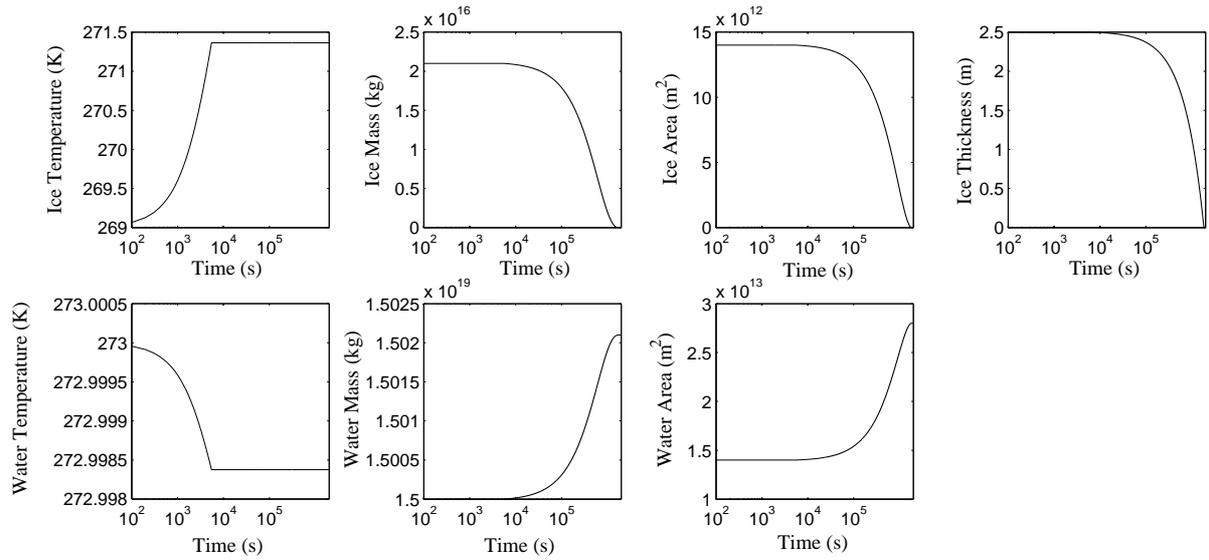


FIGURE 3. Temperature, mass, area and thickness of Arctic ice and ocean during heating and melting. The first row of graphs corresponds to ice and the bottom to ocean. The columns correspond to temperature, mass, area and thickness respectively.

TABLE 1. Test of terms in Eq. (1) and variation of parameters F_i , P and C . Numbers in brackets following the test description for tests 1 to 5 refer to the terms kept from Eq. (1), where (1) = $C(T_i - T_w)$, (2) = $F_i P$, (3) = $\epsilon_a \sigma T_a^4$, (4) = $-\epsilon_i \sigma T_i^4$.

Test #	Test Description	Time to heat and melt (months)	Difference from original time (%)
1	Basic equation – Newton’s law of cooling (1 only)	0.71	10.9
2	No emission (1, 2, 3)	0.47	-26.6
3	No atmospheric radiation (1, 2, 4)	0.83	29.7
4	No solar radiation (1, 3, 4)	1.03	60.9
5	No heat flux from ocean (2, 3, 4)	7.06	1003.1
6	Full equation (1, 2, 3, 4)	0.64	0
7	Low fraction of absorbed solar radiation ($F_i = 0.4$) [7]	0.69	7.8
8	High fraction of absorbed solar radiation ($F_i = 0.7$) [7]	0.55	-14.1
9	Low solar radiation ($P = 1000 \text{ J s}^{-1}$) [12]	0.73	14.1
10	High solar radiation ($P = 1700 \text{ J s}^{-1}$) [12]	0.57	-10.9
12	Low heat transfer coefficient ($C = 400 \text{ J s}^{-1} \text{ m}^{-2} \text{ K}^{-1}$) [6]	0.78	21.9
12	High heat transfer coefficient ($C = 700 \text{ J s}^{-1} \text{ m}^{-2} \text{ K}^{-1}$) [6]	0.47	-26.6
13	Fixed atmospheric and water temperature ($T_a = 253 \text{ K}$, $T_w = 273 \text{ K}$)	0.64	≈ 0

body emission. As z increases, the layer temperature also increases, until a maximum is reached at the ice/ocean boundary equal to the melting temperature.

At specific times, the temperature profile through the ice differs from the expected profile. The profile at 0.4 months (solid line) shows an example of this. A sharp drop to 0 K towards the lower layers of ice is seen, indicating that the melted layers of ice have temperatures of 0 K instead of the expected 273 K. This is possibly due to a numerical error or instability in the model equations.

Figure 5 shows the effect of varying the number of layers of ice. The melting process is strongly dependent on the ice layer thickness due to the assumption that

each layer melts instantaneously. The melting time is inversely proportional to layer thickness; as the thickness decreases, the melting time increases and approaches infinity for infinitely thin layers.

4. CONCLUDING REMARKS

The heat flux model based on Newton’s law of cooling resulted in the Arctic ice sheet melting in just over half a month. The improved model, based on the one-dimensional heat equation, produced a melting time of approximately one month. While this is slightly longer

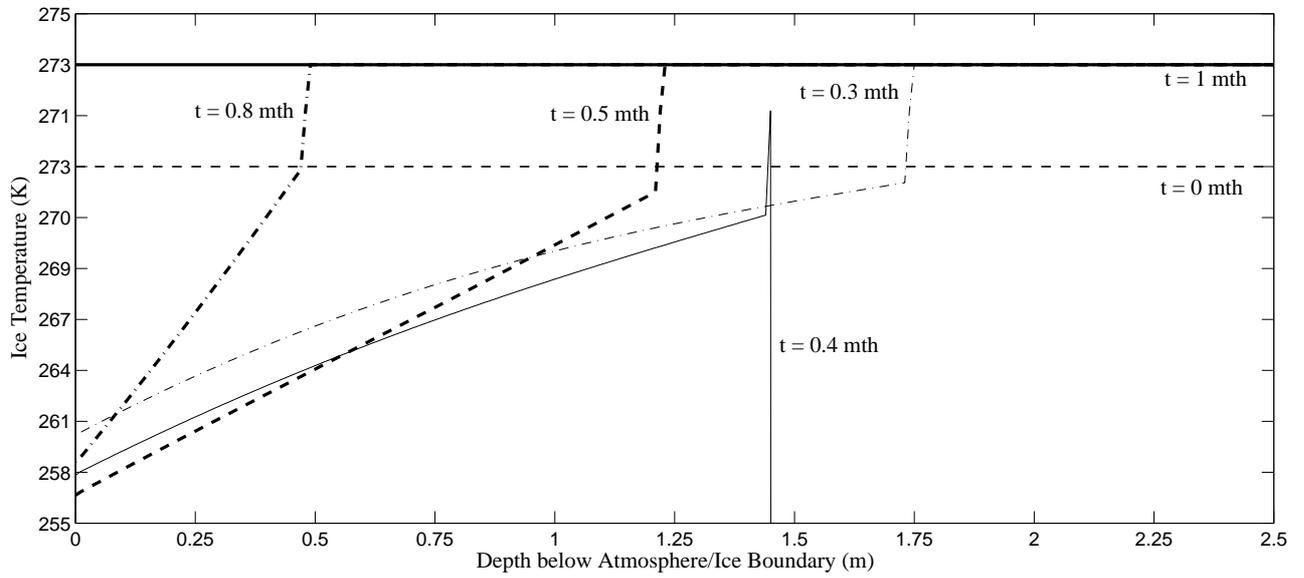


FIGURE 4. Selected temperature profiles from the heat equation model at $t = 0$ (dashed), 0.3 (dash-dotted), 0.4 (solid), 0.5 (bold dashed), 0.8 (bold dash-dotted) and 1 (bold solid) month ($\Delta t = 10,000s$, $\Delta z = 1cm$).

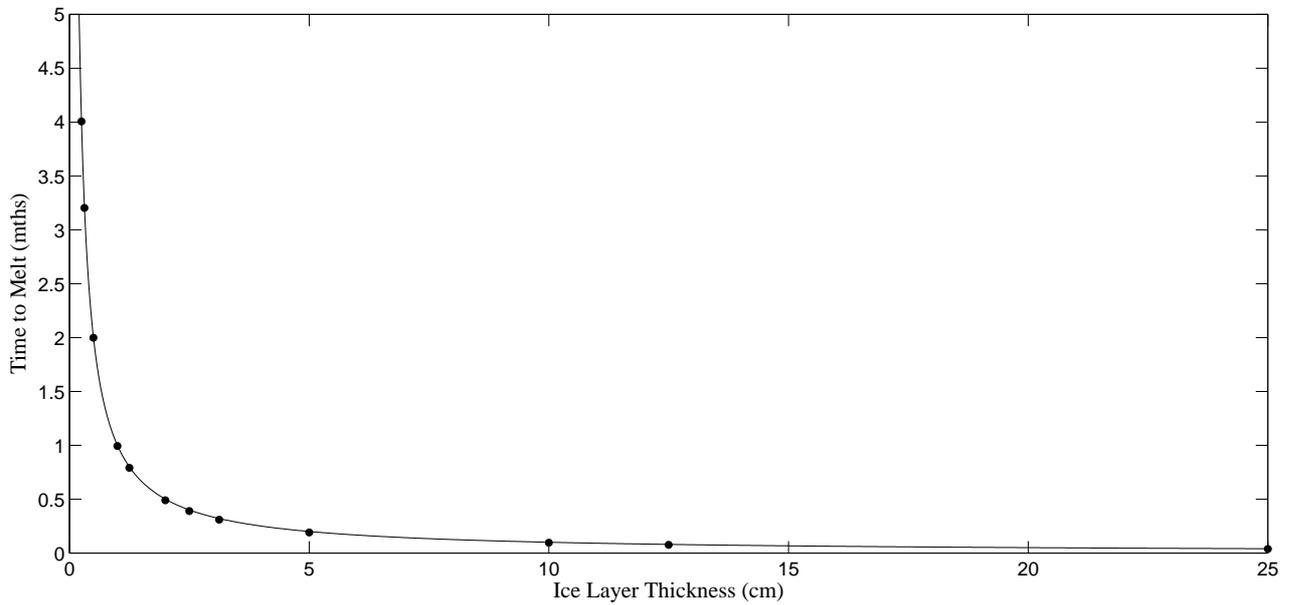


FIGURE 5. Dependence of the ice melting time on the layer thickness ($\Delta t = 10000s$)

than previously, both models imply that the Arctic ice sheet melts in the order of a month, an unrealistically short period of time. For the heat flux model, this was due to the assumption that the ice temperature was uniform throughout the ellipsoidal ice. For the heat equation model, the short melting time may be due to the assumption that the water temperature remains constant throughout the heating and melting processes. In reality, the ocean temperature near the ice/ocean boundary would change significantly during the melting process, cooling down as the ice melts.

It is difficult to compare the results produced by both models with others due to their simplicity. The no-snow model of Maykut and Unterstiner [2] predicts ice thickness to vary by no more than 35cm throughout a year, while the simplified model of Semtner [3], which includes a layer of snow, predicts a maximum variation of around 5cm. The energy-conserving model of Bitz and Lipscomb [4] predicts a change even less than this. These variations are all extremely small compared to the 2.5m predicted by the heat flux and heat equation models.

There are a number of modifications that could be made to the heat equation model to produce more realistic results. The melting process would be modelled more accurately by considering it as a Stefan problem. This would allow the position of the ocean/ice boundary (relative to the ocean floor or ice/atmosphere interface) to be determined. More detail about one-dimensional Stefan problems and their numerical solutions can be found in numerous papers, for example [13]. At the very least, the assumption that the ocean temperature remains constant with time should be discarded in favour of some function dependent on depth below the ice sheet, for example the problem studied by Gade [14].

The amount of solar radiation the Arctic receives during summer and winter months varies greatly due to its latitude; the solar radiation parameter should be time-dependent to reflect this. It would also be beneficial to allow for the ice sheet to both melt and freeze at its upper atmosphere/ice boundary as well as the lower ice/ocean boundary, similar to the way detailed by Semtner in [3] in his 3- and 0-layer models. Another modification would be to modify the fraction of absorbed radiation to be a function of ice thickness, in order to account for the important ice-albedo feedback. The presence of snow on the ice surface should also be included in the model; snow has a higher albedo (reflectivity) than ice and therefore has an insulating effect on the ice. Finally, parameters such as the heat capacity, melting temperature, thermal conductivity and latent heat depend on the salinity of the ice, and should be modified to reflect this.

In summary, additional modifications would need to be applied to the model in order for it to realistically predict the time it would take the Arctic ice sheet to heat to the melting temperature and melt completely. Although

both models are simple and reasonably fast computationally, assumptions made during their development should be revised in order for more reasonable time-scales to be predicted for the melting and heating processes.

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