

THE POPULATION GROWTH MODEL

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Abstract.

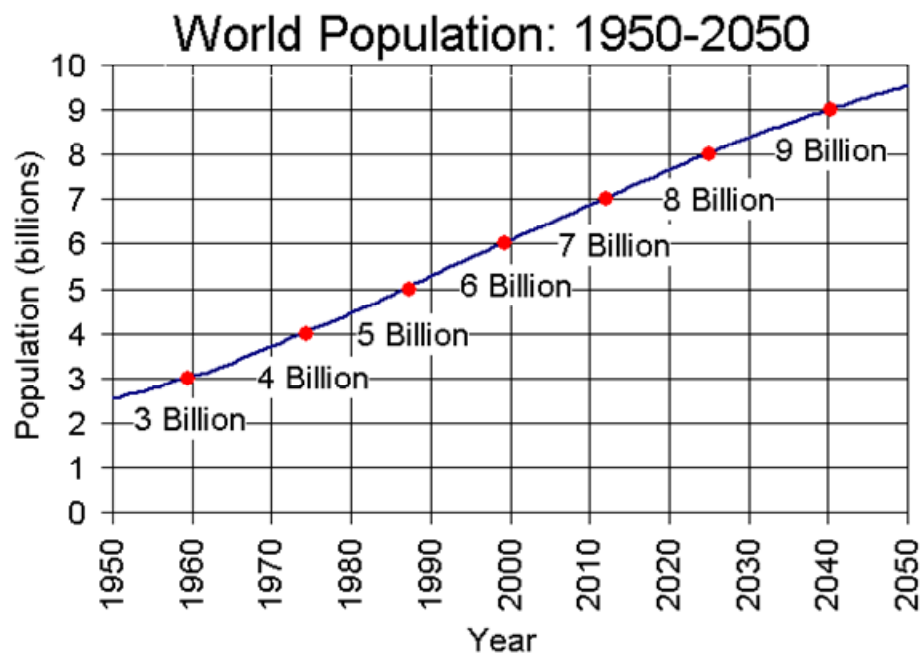
The population growth worldwide had been rapidly expanding at the unprecedented rate and had increasingly added to the demand of the food consumption, also adding economical stress on the respective countries to feed them. Historically the population growth had been an important concern to the government strategic developments and policies implementation, notably the fiscal policy and other project or endeavours that a government had to accomplish which require one vital prerequisite, which is predicting sufficient revenue. Census statistics demonstrated this importance of estimation of revenue generation and therefore influences the decision making of the federal budgets."The importance of population growth rate lies partly in its central role in forecasting future population trends; indeed if the form of density dependence were constant and known, then the future population dynamics could to some degree be predicted. The population growth rate and its dynamic models are fundamental to our understanding of environmental pressure/stress. Dynamic models are progressively more used in various academic fields ranging from mathematic, social science to economics, particularly in macroeconomics. "Ever since Malthus An Essay on the Principle of Population, there has been a vigorous public debate concerning various models of population growth, and the relationship between different forms of population growth and various kinds of biological and cultural phenomenon (Robert 1980).In the present investigation we explore the dynamic context of population growth model using these variables: the birth rate, death rate, pollution, natural resources and the capital investment.

Introduction.

Historically, Botero in 1588 was among the earlier pioneer to suggest the carrying capacity as restrain to population growth a rate, Malthus elaborate this ideas and put forward the mathematical analyses which Malthus essay on the principle of the principle of population (1798)."Malthus's Principle of Population provides a natural context for introducing dynamic analysis. Moreover, the context is useful for reviewing many mathematical tools and theoretical constructs often used in economics" (Mark 2003). we assumed that population growth is determined and underpin by the shortage of natural resources which act as carrying capacity and a hinderer to the volatile and unstable proliferation. However, loyalty to dynamic polices and population fluctuation behaviours for the period of successive phases of their growth cycles, as results of subjection of unpredictable forces and also the closeness of gap between the populations to the carrying capacity. So "trying to estimate the carrying capacity for a given situation seems a little like searching for the Holy Grail" (Robert.1980).the common postulation is that "While a population is far below its carrying capacity, it is said to grow exponentially and to be subjected to "r-selection." Later, as the effects of population pressure accumulate, the forces of "K-selection" begin to predominate, and population growth often levels off logistically (Robert 1980)". The population growth in reality is of what obscure as there is strong propensity to overly populate which overwhelms the ability of natural resources or productivity, this was first recognise by Malthus as he continued "The power of population is indefinitely greater than the power in the earth to produce subsistence for man. This led us to uses estimate computation within the range that does not completely deplete natural resources. Here in present dynamic model, several variables and vitals forces and above assumptions have been examine on how they affect each other and ultimately producing the total consequences that we watch. We explore the

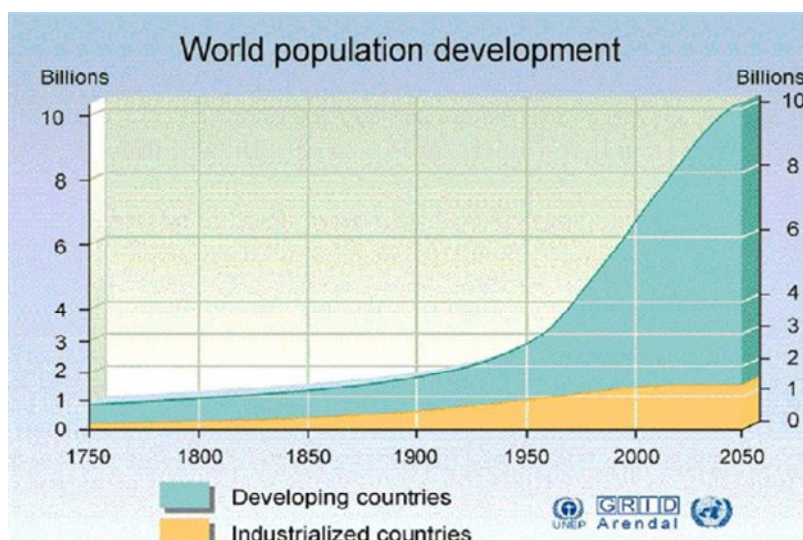
population growth in the dynamical nature/context. Most importantly considerable's variables include: the birth rate, death rate, pollution, natural resources and the capital investment.

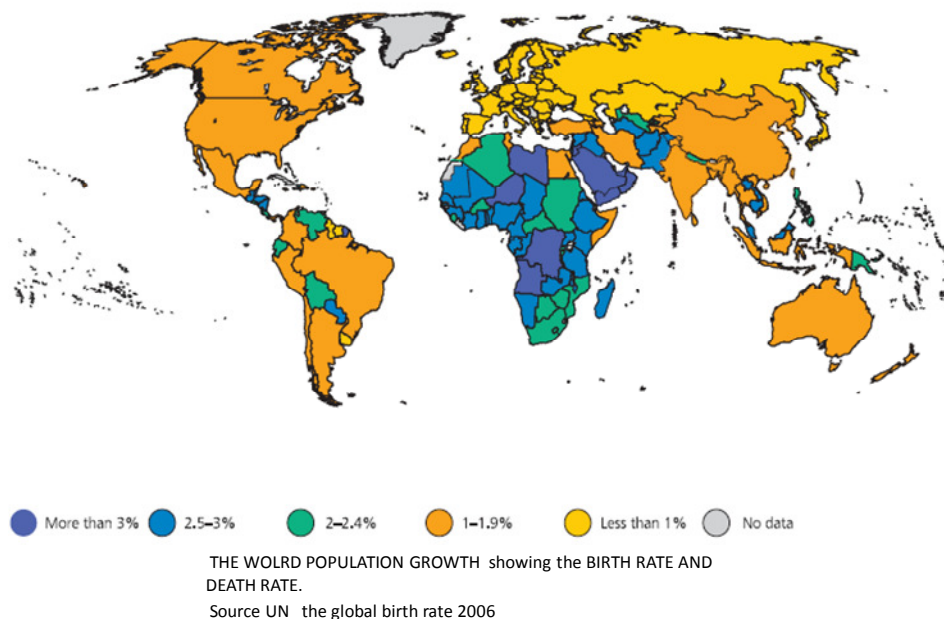
THE TREND (alarming problem).



Source: U.S. Census Bureau, International Data Base, December 2008 Update.

Source: United Nations, "The 2006 Revision Population Database"





The above diagrams both point to one thing, the alarming rate of the global population growths, “The world population increased from 3 billion in 1959 to 6 billion by 1999, a doubling that occurred over 40 years. The Census Bureau's latest projections imply that population growth will continue into the 21st century, although more slowly. The world population is projected to grow from 6 billion in 1999 to 9 billion by 2040, an increase of 50 percent is expected to require 41 years”(US census bureau). However its quiet easy to identify the variation in the population growth, the developing countries is the mains contributors of the largest percentage proportion to the population growth. This variation arises from the realistic and complicated factors ranging from socioeconomic, cultural, religion and political ideology(system and practises) and stipulation just to mention a few, which individually and collectively contributed to the overflowing of the population making it hard to identify the individual factors contribution. Speaking in general terms in regard to the demographic transition, it’s had started earlier in the developed countries in comparison to developing countries and virtually there is no progress in other countries in fact they are getting worse as the globalisation and technological development increases. With this scenario at hand then we devise a dynamic models to tells us what does that increase in the population means to our lifestyles and how can we cope or is there any possible policies that can be pursue/practice to get us out of the pit.

MODEL AND ASSUMPTION.

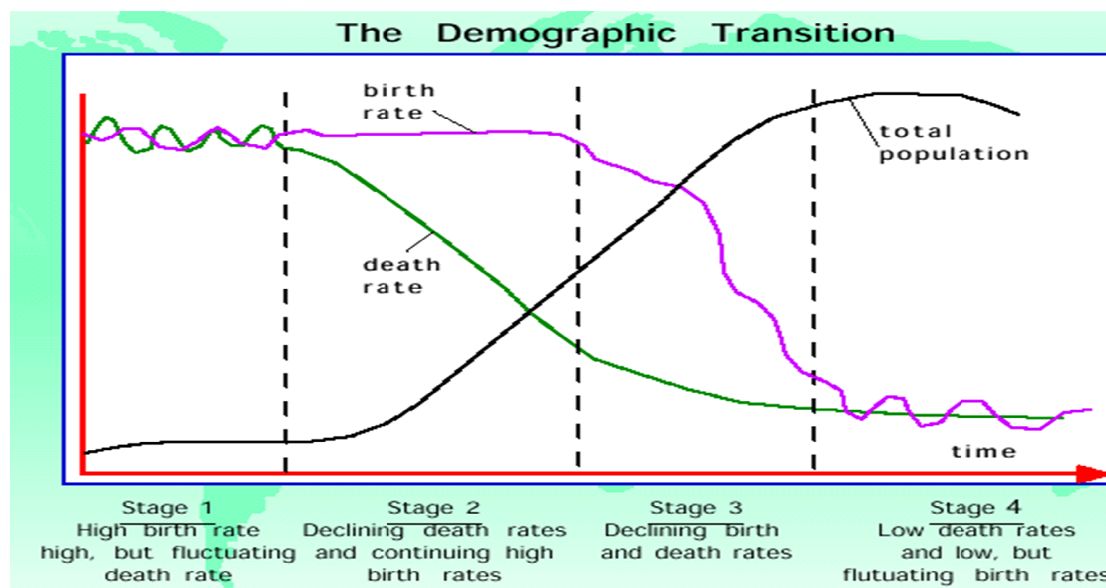
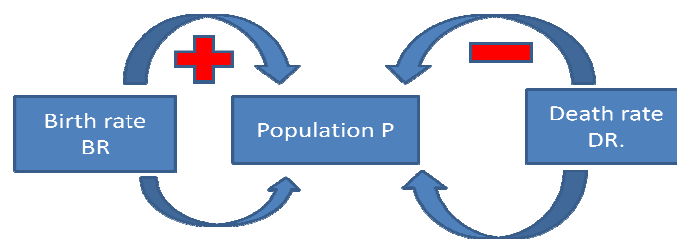
The dynamic interaction of the world population growth and it variables involves most controversy and complexity beyond this paper. Here we try just to skim through the layer of this complexity, starting building the model from basic and gradually increase the complexity as more and more variables are added.

Here is two majors' feedback loop that fundamentally affects population growth. The birth loop which add to population and the death loop which deplete the populations and are measured in thousand people per a year.

Majors factor that affects population growth:

- The birth rate (α) which add to population
- the death rate (β) which deplete the populations
- carrying capacity (P_0), determined by the amount of natural resources

$$\frac{dP}{dt} = (\alpha - \beta)P \left(1 - \frac{P}{P_0}\right)$$

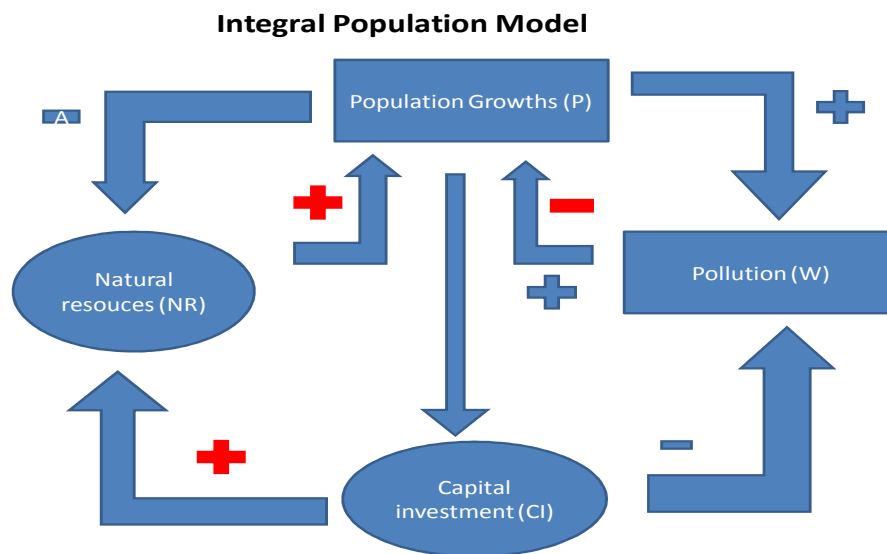


Source: www.globalchange.unic.edu

FIGURE 1 the demographic Transition: Showing the changes in EUROPE.

The combination of decreasing death rate due to the much of progress in sanitation and medicine, coupled with the decrease in birth rate due to changes in the economies, has led to a profound change in the population growth curve in the developed world.

This figure reinforces the above equation showing the pooling of population being made up of the fertility and mortality rate. This Demographic Transition Model is a simplified version of reality that shows population change (births, deaths and natural change) over time.



Dynamical Model for population

$$\frac{dP}{dt} = (\alpha - \beta)P \left(1 - \frac{P}{P_0} \right)$$

$$P_o = aN$$

$$\frac{dN}{dt} = N_0 - \phi P$$

$$\beta = \beta_0 + fW$$

$$\gamma = \gamma_0 - eC$$

$$\frac{dW}{dt} = \lambda P - \gamma WC$$

$$C = \rho P$$

P = Population

N = Natural Resources

W = Pollution

C = capital investment per year

Above are the schematic diagram and differentials equations showing the feedback relationship and effects of the individual variables.

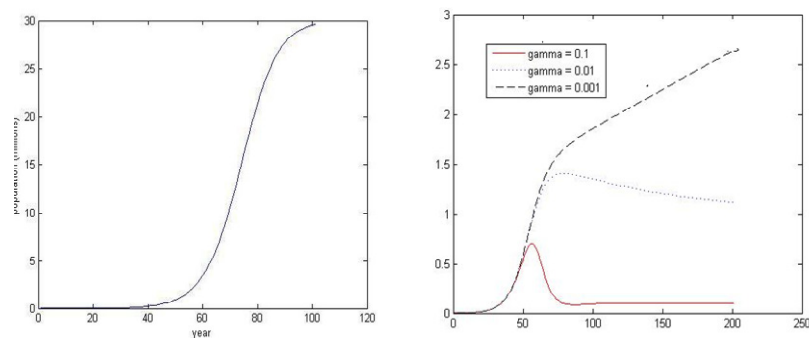
The first equation imply that population growth is made up the birth, death rate and the carrying capacity.

The population growth reduces the level of the natural resources stock especially under assumption of non-renewable resources stock, Whereas the natural resources act as the carrying capacity in determining the population growth (po). “As the population rises, the rate of usage of natural resources increases and the remaining store of resources shrinks to collide with the rising demand” (Jay .w.) The diminishing of the natural resources could also affect the population growth as the economist Malthus had argued that is indicated by red plus meaning those effects are not considered or factored in the model.

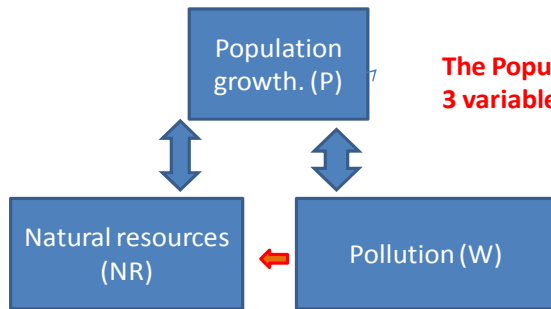
The population growth contributed to the capital investment through the means of raising revenues by employments and taxation. The capital investment is assumed to have positive effects on the population growth via technological progression and improvements of living standard. As the population growth then government receives more revenues of which if invested wisely in the productivity- enhancing investments process, it could alleviate the shortcoming and therefore lead to more output in the long run as assumed ,thus the more population growth the more capital investment is exhibited That is reho in the CI equation. Population growth could increase pollution levels as the productions are boost up to keep up with the demand. However the rise in the pollution levels is detrimental to population growth and lead to an exaggerated death rate reducing the population levels which is denoted (fW) in the death equation plus the initial death rate. Below we explore and computer statistical data using MATLAB to investigate these population parameters.

RESULTS.

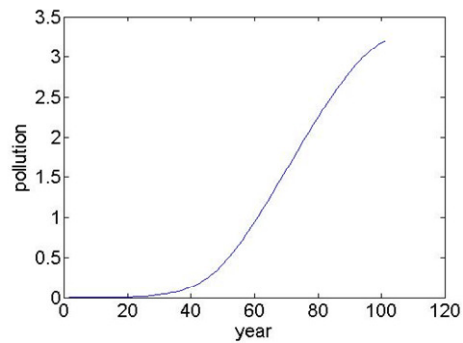
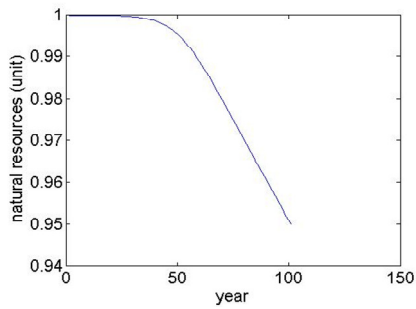
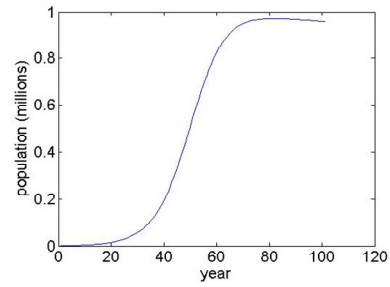
Exponential population grow and the population effects on the natural resources – the gamma affects



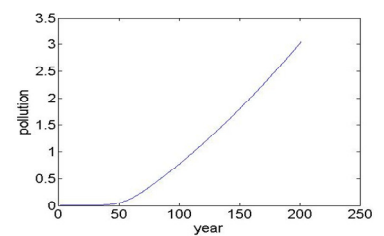
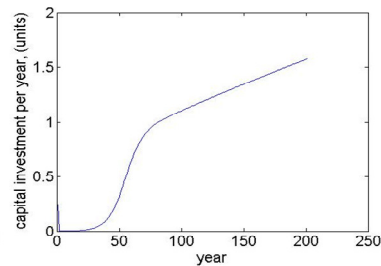
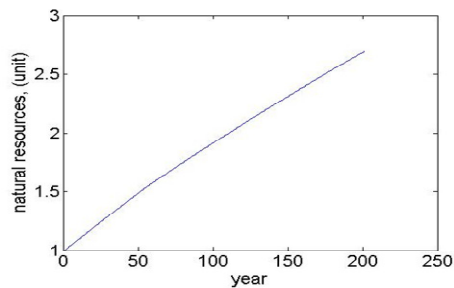
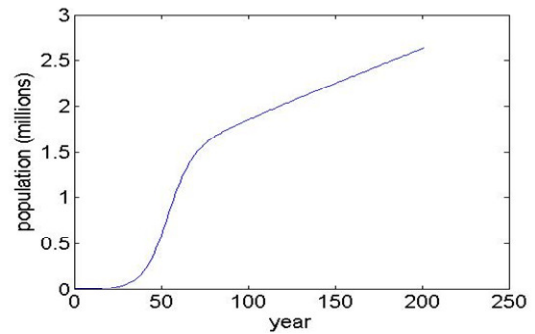
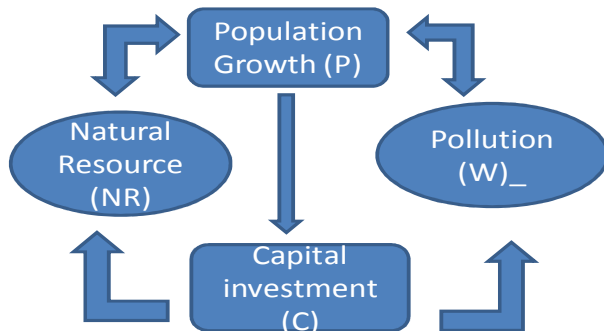
The diagrams above depict and confirm our assumptions meaning the observed or produced diagrams were expected. The sigmoidal shape of the population growth is the hall mark of exponential population growth. Secondly the gamma affects was clearly highlighted, as the population grow this non-renewable resources store are rapidly depleted.



The Population growth and its interactions with 3 variables



- The population growth and its interactions with 4 variables.



The results when more parameters are factor in are amazing, first was the under the non-renewable resources population face extinction after the resources depletion, however now we uses renewable resources and the natural resources increases at the decreasing rate and amazingly enough its doesn't experiences immediate depletion, in the three variable diagrams we see the effects of the pollution on the population as the pollution peaks its immediately have consequential effects on the population growth shown by the flattening out of the population. we assumed that population growth is determined and underpin by the shortage of natural resources and pollution as Malthus has assumed, this evidently suggest to us that exponential growth cannot persist indefinitely. In comparison the last diagram had pronounced effects as both the pollution and the natural resources started biting shown by the notch in the population but luckily capital investment came to the rescued and if it followed the assumption layout above then it should really stabilise the systems, however something astonishing and unexpected happen. if you look at the natural resources figure, its show an increases as the population growth amplify suggesting capital investment actually creates some natural resources or more rationally ,that as we become more efficient especially with the renewable resource usage , they can be use plentiful times consequently amplifying natural resources beyond there initial quantity.

CONCLUSION.

The population growth dynamic model had shown its usages and its degrees of predictability, its reliability thought is another entirely different question Methods often are questionable as in most research in population growth, so it would require rigorousness repetition and reliable date . These illustrative diagrams are not believed an ideal representation of the linkages; conspicuously its superficiality or it lacks of explicit linkages among population growth and it variables "Population dynamics, natural resources and environmental change are linked in many ways and through multiple social and economic mechanisms, at various geographic levels. But not all those linkages have relevance for policy formulation in one of the three domains thus interconnected. This paper tries to identify policy issues among the array of conceivable linkages, placing emphasis" (Marcoux 1999) on population effects on the environment and the capital investment in a general way. However despites those limitation, this dynamic model had produces some significant progress in its predictability, it shows how capital investment can be our future hope and suggest more focuses on the efficient usages of renewable resources and identify the pollution as a human main nemesis." This perspective emphasizes the role of technological change in enabling adaptations, and therefore in accommodating more pollution and population growth" (Marcoux1999). It shouldn't be view that technology progress merely **buys time** or as a temporary remedy which would tie our hand and lead us ultimately to limited capacity to solve unpredicted future problems. However it left us with one action to mobilize ever more technological ingenuity, renewable resources and advocated the environmental policies that enabled us strive ahead of the pollution e.g. massive technological advancement to enhance in the dissipation of pollution into some harmless and inactive form and perhaps stable population growth.

References

1. Jay W. Forrester .World Dynamics (1971).
2. Robert .m. Schacht the two dynamic models of population growth, *American Anthropologist*, Vol. 82, No. 4 (Dec., 1980), pp. 782-798.
- 3 Marcoux. A. Populations and Environmental Change from Linkages to Policy, Population Programme Service (SDWP), Issues (1999)
- 4 U.S. CESNSUS BUREAU INTERNATIONAL DATA BASE, DECEMBER 2008 UPDATES.
5. United nations the 2006 revision population database
6. United Nations gobal birth rate 2006.
7. Partial Differential Equations: Modelling and Numerical Simulation (Computational Methods in Applied Sciences) by Roland Glowinski, Pekka Neittaanmäki (2008)
8. Richard M. Siblyl and Jim Hone Population Growth Rate and Its Determinants, *Biological Sciences*, Vol. 357, No. 1425, 1999
9. Pingle. M. Introducing Dynamic Analysis Using Malthus's Principle of Population, the *Journal of Economic Education*, Vol. 34, No. 1 (winter, 2003), pp. 3-20