Investigation of the Critical State in soil mechanics using DEM

Andrés A. Peña*, Ramon García-Rojo†, Fernando Alonso-Marroquín** and Hans J. Herrmann‡

*Bilfinger Berger GmbH, Gustav-Nachtigal 3, 65915 Wiesbaden, Germany;
Institute for Computational Physics, « University of Stuttgart », Pfaffenwaldring 27, D-70569 Stuttgart, Germany
†EPCOS, Avda. Jose Ortega y Gasset 173, 29006 Mï£¡laga, Spain
**School of Physical and Mathematical Sciences, The University of Queensland, St. Lucia, Qld. 4067 Australia
‡ Departamento de Física, Universidade Federal do Ceará, 60451-970 Fortaleza, Ceará, Brazil;
Computational Physics, IfB, HIF E12, ETH Hönggerberg, CH-8093 Zürich, Switzerland

Abstract. The existence and uniqueness of the so-called critical state in soil mechanics is validated in our DEM simulations of irregular polygonal particles. « For different particle shape characteristics„ The critical state is found to be independent of the initial stress and density.» We retain low stress levels, since we do not take into account the crushing of particles. « »In biaxial test simulations isotropic particles evolve toward a limiting state in which the system reaches a critical void ratio and deforms with constant volume, deviatoric stress, fabric anisotropy, and mechanical coordination number. The last one has been found to be the first variable to attain a critical value making possible for the rest of micro-and-macro-mechanical variables the convergence to the critical state. In periodic shear cell tests, for large shear deformations samples with anisotropic particles reach at the macro-mechanical level the same critical value for both shear force and void ratio. At the micro-mechanical level the components of the stress tensor, the fabric tensor and the inertia tensor of the particles also « reach » the same stationary state. By varying the aspect ratio of the particles we stated the strong influence of particle shape anisotropy on the parameters that the granular packing « » at the critical state.

Keywords: Critical state, fabric anisotropy, particle orientation
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1. INTRODUCTION

The stress-strain behavior of both dense and loose sand under shearing is first described by Casagrande in 1936 [1]. He concludes that sand for large shear deformations independent of the initial density state reaches a limiting state (critical void radio) in which samples undergo unlimited deformation without further volumetric and stress increments. He also finds that the critical void ratio is only dependent on the confining pressure, and thus determines the so-called critical state line relating the critical void ratio e_c and the effective normal stress σ' as applied on the shear box test.

The existence and uniqueness of this critical state is a major feature in soil mechanics since it is used to define post-failure behavior of many constitutive models describing granular materials [2, 3, 4, 5, 6, 7]. The « » critical state has been experimentally proven to be independent of sample preparation and test conditions [8, 9]. Nevertheless, since there are some experimental difficulties to characterize the pre-and-post peak mechanical behavior of dense samples arising from the strain localization [3, 10]. There are some groups of researchers that claimed that the uniqueness of this state is still an open issue [11], and depends on the consolidation history of sand specimens [12]. «The question of uniqueness is essential in the investigation of granular materials at large shear deformations, as is observed in fault gouges [13]. « The limit state has been investigated using DEM simulation of biaxial test [**?**] and simple shear test [13]. An important step towards the development on the micromechanical continuum modeling of granular materials for large shear deformation is to determine how this state depend on the initial conditions and the granulometric properties of the granular assembly [**?**] »

In this work, we investigate « the critical state » by means of numerical simulations of polygonal packings of particles. The samples consist of isotropic and anisotropic particles in order to study the influence of particle shape anisotropy on the global mechanical response of granular media and its evolution to the critical state. We show that the DEM simulations reproduce the main features of the critical state in soil mechanics, namely, the granular media evolve toward a stationary state in which the system reaches a constant void ratio and deforms with constant volume and deviatoric stress, and that for different initial stress states the corresponding stationary values collapse onto a unique critical state line. The model is described in detail in [14].

2. EXISTENCE AND UNIQUENESS

In order to assess the existence of the critical state, we first explore the macro-mechanical evolution of granular samples under biaxial compression. We characterize the density state of the samples by the void ratio e.

The macro-mechanical response of the granular media is presented in Figures 1(a) and 1(b). The evolution of $\sin\phi = (\sigma_1 - \sigma_2)/(\sigma_1 + \sigma_2)$ with axial strain ε_1 for the dense and the loose samples is presented in Figure 1(a). « Dense samples exhibit a peak and strain softening behavior, whereas in loose samples no peak is observed » . However, both systems at large shear deformations present a tendency to stabilize around a value that one could consider as the steady state of the material $(d\dot{sin\phi}/dt = 0)$.

The evolution of the void ratio with axial strain is illustrated in Figure 1(b). The dense samples initially contract and later dilates. The loose samples contract. For large axial strain values the void ratio reaches a constant value. The void ratio in both dense and loose samples varies until it achieves a constant value between 0.23 and 0.26. This stationary value of e is slightly different for each sample, since the parameters e and ϕ at this stationary state depend on the granulometric properties of the material [15, 16]. In this stage of large deformations, the granular medium is deformed at constant volume and with the same approximate value of deviator stress. This state corresponds to the critical state of the material and it is independent of the initial sample density [2].

Another issue we address is the evolution of the anisotropy of the contact network of the granular packing [17, 18]. « This » is characterized using the deviatoric component of the fabric tensor $\mathbf{F}$ which takes into account the orientational distribution of contact normal vectors $\vec{n}$. In Figure 2 a the evolution of the deviatoric component $F_{11} - F_{22}$ of the fabric tensor with ε_1

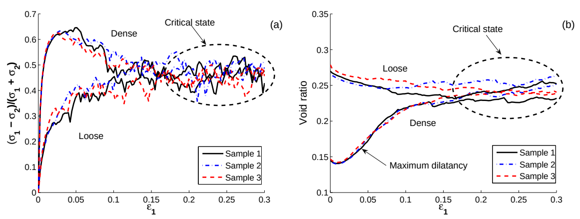

FIGURE 1. Evolution of (a) the deviator stress and (b) void ratio of the samples used to asses the existence of the critical state. Simulation parameters, $p_0 = 64$ kN/m, $N = 900$ particles and $\mu = 0.5$. Samples correspond to different seed for sample generation.

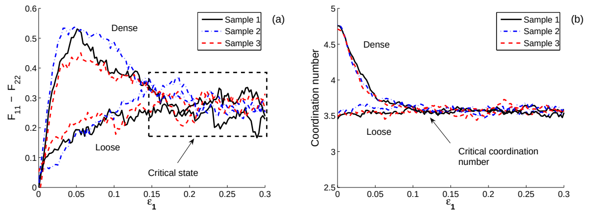

FIGURE 2. Evolution of (a) the deviatoric component $F_{11} - F_{22}$ and (b) the trace $F_{11} + F_{22}$ (coordination number) of the fabric tensor of the samples. Simulation parameters, $p_0 = 64$ kN/m, $N = 900$ particles and $\mu = 0.5$.

is presented. One can notice that the contact network starts from a rather initial isotropic configuration $F_{11} - F_{22} \approx 0$, and that as soon as the shear process begins anisotropy is developed. This anisotropy is a result of the creation and reorientation of contacts and force chains along the direction of loading. For the dense samples, the anisotropy increases until the granular system develops its peak strength i.e. the maximum anisotropy coincides with the maximum strength (≈ 5 % axial strain ε_1). On the other hand, the anisotropy in loose samples simply increases until it saturates at a constant value between the statistical fluctuations. In the strain-softening regime the anisotropy of dense samples decreases until it converges to the same value that the loose samples have reached. Hereafter, the media deforms at a critical anisotropy. This has been previously observed in numerical simulations of biaxial tests with the DEM [19, 20, 21].

The creation and destruction of contacts can be studied by following the coordination number Z. The trace of the fabric tensor $\mathbf{F}$ gives the coordination number. « In Figure 2b we present the evolution of the coordination number of the samples ». At low axial strain values, the dense system contracts and as a consequence a small increment of Z is observed. This is followed by a decrease

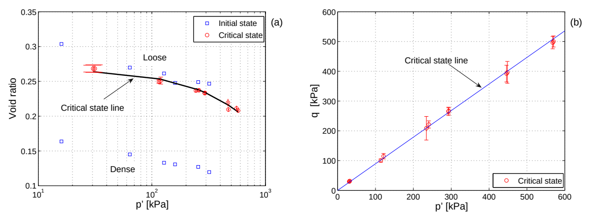

FIGURE 3. Critical state line in (a) the compression plane void ratio e - mean stress p', (b) the stress plane q - p', and (c) the deviatoric fabric $(F_{11} - F_{22})$ - mean stress p' plane. System parameters, $N = 900$ particles and $\mu = 0.5$. In (a) the squares indicate the initial state of the samples. The circles are the values that samples reach at the stationary state, and the error bars correspond to 1 standard deviation of the analyzed data.

of the Z value when the system start to dilate. This decrease is associated with the the breaking of interlocking between particles and the related formation and collapse of force chains along the direction of loading. As a result, each particle begins to lose contacts. Both samples around 8% axial strain reach a similar coordination number Z close to 3.6. This critical coordination number is the first signal that the granular packing is evolving towards the critical state, and at the same time it enables the contact network to reach an steady structure.

We also perform biaxial experiments using different initial confining pressures p_0 in order to assess the uniqueness of the critical state, i.e., that there is a unique void ratio for each state of effective stress at the critical state. Since our model does not take the crushing of particles into account, we retain low stress levels.

In Figure 3a, we plot in the compression plane (void ratio e - mean stress p' $(p' = (\sigma_1 + \sigma_2)/2)$) the initial states of the samples (blue squares) and the values of void ratio that the loose and dense samples attain at large deformation (red circles). One can see that the same limit state is reached for all the samples defining a unique steady state line. This line can be fitted by a function of the form,

$$e(e_{ref}, n) = e_{ref}\ exp(-n\ (p'/p'_{ref})), \tag{1}$$

where e_{ref} and p'_{ref} are the void ratio and the mean stress at the critical state corresponding to the reference initial confining pressure « $p_{0,ref} = 16kN/m$ », and n is a constant. The same collapse of the stresses at the critical state (red circles) onto a steady state line is observed in Fig. 3b, in which the stress plain $q - p'$, being $q = (\sigma_1 - \sigma_2)$, is depicted. The stress ratio q/p' at the critical state defines the strength parameter M, which for our simulations is related to the critical state friction angle ϕ_{cs} as,

$$M = \frac{q_{cs}}{p'_{cs}} = 2\ sin\phi_{cs}\ . \tag{2}$$

The range of variation of the critical friction angle found in our simulations, i.e. $22^\circ \leq \phi_{cs} \leq 27^\circ$, is smaller than the limits usually obtained for sand and gravel in three dimensional experiments in realistic soils 26° to 36° [9, 16]. This is explained in terms of the higher coordination number of systems in 3D [22], specifically, the higher the coordination number the higher the strength of the material as presented in Figures 1a and 2c.

The previous simulation results support the idea of uniqueness of the critical state [2, 3], in which a critical/steady state line links the critical states describing combinations of effective stresses and void ratio $e : q : p'$.

Finally, we also evaluate the critical anisotropy for different stress sates, and we find that a critical state line for anisotropy can also be defined. Although no presented, the relation between structural anisotropy and mean stress p' is best fitted by a linear function.

Contrary to samples with isotropic particles, the samples with anisotropic particles do not converge to the critical state under biaxial compression. This is explained in micro-mechanical terms as follows: whereas the isotropic samples reach a critical value of anisotropy and coordination number, the contact network of anisotropic samples is still changing. The non-stationary state of these variables is directly related to the evolution of the particle orientation. Elongated particles are reoriented during the shear process without converging to a steady state. This micro-mechanical evidence, concerning the non-stationary state of the fabric and particle orientation, does not allow the systems to reach the critical state [14].

3. INFLUENCE OF ANISOTROPIC PARTICLE SHAPE

In this section, we study the existence of the critical state for samples consisting of anisotropic particles by means of shear cell experiments. For this experiment, periodic boundary conditions are imposed in horizontal direction. The top and bottom have fixed boundary conditions. The top and bottom layers of the particles are moved in opposite directions so as to impose a constant shear rate $\dot{\gamma}$. Two different initial configurations are obtained for the anisotropic samples, (i) the grains are oriented parallel to the shear direction (called "horizontal" sample - H), (ii) the grains are oriented perpendicular to the shear direction (called "vertical" sample - V).

We find in our simulations that both the ratio of shear force to normal force F_s/F_n and the void ratio e evolve toward the same saturation value independently of the initial anisotropy due to contact and particle orientations. Anisotropic samples also reach the critical state.

At the micro-mechanical level the deviatoric component of the fabric tensor **F** and inertia tensor **I** also reach a stationary value independently of the initial particle orientation. The stationary value of the deviatoric component is directly related to the steady state at the macro-mechanical level, and seems to be a micro-mechanical requirement for the global steady state.

Although the results discussed herein and presented in [14] correspond to dense samples and one value of confining pressure, they are valid for different initial density states and stress levels. These results validate in our MD simulations the existence of the so-called critical state in soil mechanics irrespective of any initial condition and particle shape characteristics.

Furthermore, we consider dense samples with elongated particles initially oriented in the vertical direction and with the aspect ratios $\lambda = 1.0$, 1.5, 2.3, 3.0 and 4.0. The confining pressure p_0 is kept constant at 16 kN/m, and the shear rate $\dot{\gamma} = 1.4$ s^{-1}.

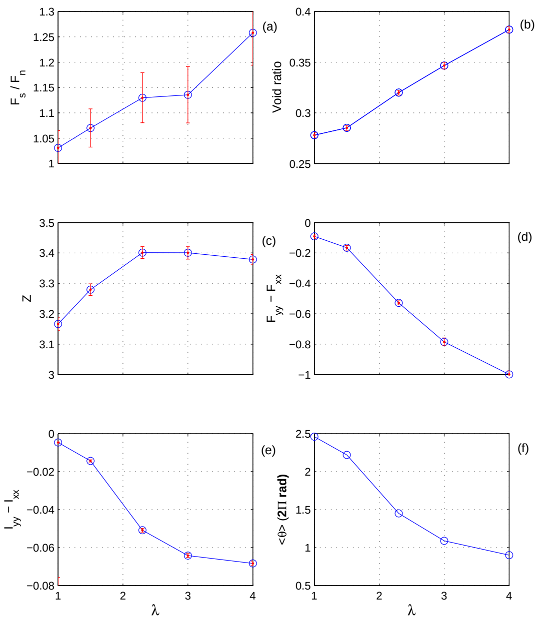

FIGURE 4. Effect of particle shape on the critical state values attained by the granular packings on shear cell tests, (a) ratio shear - normal force F_s/F_n, (b) Void ratio, (c) coordination number Z, (d) deviatoric fabric $F_{yy} - F_{xx}$, (e) deviatoric inertia $I_{yy} - I_{xx}$ and, (f) mean accumulated rotation $\langle\Theta\rangle$ of the particles since the beginning of the simulation till a shear strain $\gamma = 30$. The following aspect ratio λ are used: 1.0, 1.5, 2.3, 3.0 and 4.0. The shear rate $\dot{\gamma} = 1.4\ \mathrm{s}^{-1}$. The error bars correspond to 1 standard deviation of the analyzed data.

In Figure 4 we present the average values of the macro and micro-mechanical parameters for the different aspect ratio at the critical state. We consider the ratio between the shear and normal force F_s/F_n, the void ratio e, the coordination number Z, the deviatoric component $F_{yy} - F_{xx}$ of the fabric tensor **F**, the deviatoric component $I_{yy} - I_{xx}$ of the inertia tensor **I**, and the mean accumulated rotation of the particles $\langle\Theta\rangle$. The data correspond to the average of the variables once the critical state has been reached. The standard deviation is also presented. The evolution of the deviatoric components $F_{yy} - F_{xx}$ and $I_{yy} - I_{xx}$ with shear strain γ is also shown in Fig. 5. From Figures 4 and 5 we conclude the following:

- The larger the anisotropy of particle shape λ, the larger the strength of the material at the critical state (Fig. 4a).
- The larger the anisotropy of particle shape λ, the larger the void ratio at the critical state and, therefore, the larger the volumetric deformation (Fig. 4b).
- For $\lambda \leq 2.3$ the larger the anisotropy of particle shape λ, the larger the coordination number Z of the particles. For $\lambda > 2.3$ the Z value saturates and remains constant (Fig. 4c).
- The larger the anisotropy of particle shape λ, the larger the fabric anisotropy at the critical state (Fig. 4d).
- The larger the anisotropy of particle shape λ, the larger the anisotropy related to particle orientation at the critical state (Fig. 4e).
- The larger the anisotropy of particle shape λ, the smaller the accumulated mean particle rotation angle $\langle\Theta\rangle$ (Fig. 4f).
- The larger the anisotropy of particle shape λ, the longer the time to reach micro-mechanical equilibrium in fabric and particle orientation (Fig. 5).

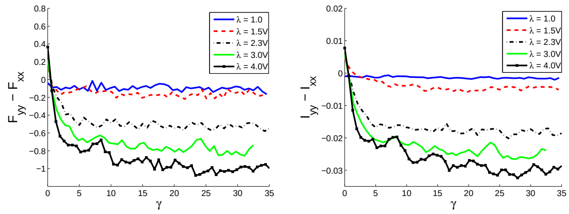

FIGURE 5. Evolution of the deviatoric component of the (a) fabric tensor **F** and (b) inertia tensor **I** for isotropic particles ($\lambda = 1.0$), and elongated particles ($\lambda > 1.0$). Samples with anisotropic particles are initially oriented in vertical direction (V).

The above statements concerning the influence of anisotropic particle shape on the macro-mechanical behavior of granular packings at the critical state, specifically, larger mobilized shear strength and more sensitivity to volumetric changes (dilatancy) with the increment of the aspect ratio λ, are explained in terms of the bigger interlocking among particles and the strong frustration of rotation that such particles undergo during shearing. Particle rotation is further hindered by the larger coordination number that anisotropic particles develop due to the larger relative flat surface. The last contribution to the macro-mechanical observations is the larger structural anisotropy (fabric) attained by the anisotropic systems at the critical state.

4. CONCLUDING REMARKS

« The steady state that granular materials reach under larger shear deformations have been assessed for different initial conditions using isotropic and anisotropic particles. The existence and uniqueness of the critical state is verified for isotropic shaped particles using biaxial test simulations. Contrary of that, samples with anisotropic particles do not reach the critical state under biaxiaL test, but they do reach this state in the periodic shear test. » Furthermore, the samples deform at constant void ratio, shear stress, fabric anisotropy, particle orientation and mechanical coordination number. The last one has been found to be the first variable to attain a critical value making possible for the rest of micro-and-macro-mechanical variables the convergence to the critical state. The uniqueness of the critical state is validated in our simulations, when it is found that the critical states related to different stress states collapse onto only one critical state line.

« In the simple shear test », samples with anisotropic particles reach the same saturation value in the steady state independently of the initial orientation of the particles. This is related to the removal and reorientation of the initial inherent anisotropy (fabric and particle orientations) in the direction of the induced shear. It is found that a steady value of fabric anisotropy and particle orientation is a micro-mechanical requirement for the existence of the critical state.

Finally, by varying the aspect ratio λ of the particles we can state the important effect of particle anisotropy on the macro and micro-mechanical response of granular media. « The evolution of the contact network, which is characterized by bucking of force chains and stress drops events, show a strong dependency on the particle anisotropy, which is still under investigation. Another important question to expore in the near future is whether the critical state depends on the loading history of the material.»

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