Micromechanics of shear bands in granular media

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The aim of this work is to study the strain localization and the fluctuations in shear bands using molecular dynamics simulations. We show that: (1) The localization of frictional dissipation acts as a precursor mechanism of failure; (2) Buckling of strain columns may explain the finite width of shear bands; (3) The building and collapse of stress columns lead to stress and dilatancy fluctuations in the critical state; (4) Vorticities in the displacement field assisted by rolling between the grains reduce the dissipation with respect to the expected value of simple shear. The implications of these rotational modes in the Cosserat theory and in the study of tectonic activity are outlined.

1 INTRODUCTION

The understanding of localized failures in granular materials in the evaluation of natural hazards is growing in importance in this day and age. Catastrophic events such as earthquakes, landslides or snow avalanches produce mass displacements large enough to devastate entire villages or towns. Typically these deformations are localized in thin layers which are called *shear bands*.

Traditional continuum theories describe shear bands as interfaces along which solid masses move like rigid blocks sliding against each other. Many physical effects are neglected in this description. For example, measures of seismic activity and heat flow in the San Andres fault show areas extended over hundreds of kilometers, called *seismic gaps*, where the current earthquake activity is quite lower than in the past (Mora and Place 1999). As an explanation of these gaps, it has been proposed that the material in shear bands organizes itself in such a way that it acts as a gear mechanism (Mahmoodi-Baran, Herrmann, and Rivier 2004). The rolling between the grains may explain the lack of heat production and earthquakes.

In this contribution we present a micromechanical investigation of the formation and dynamics of shear bands using discrete element modeling. In Section 2 we focus on the micromechanical description of strain localization and the thickness of shear bands. The stress and dilatancy fluctuations in shear bands and the role of the rotational modes in the heat dissipation is discussed in Section 3.

2 SHEAR BAND FORMATION

We study the strain localization by using granular dynamics simulations of polygonal packings. The polygons interact via contact forces with a normal and a tangential stiffness of $k_n = 160MPa$ and $k_t = 0.33k_n$, and a Coulomb friction coefficient of $\mu = 0.25$. The boundary conditions are chosen in order to mimic the experimental tests under plane strain conditions: First, a confining pressure is applied to the sample through a flexible membrane. Then, two horizontal walls at the top and bottom of the packing are used to apply vertical loading with constant velocity (Alonso-Marroquin 2004).

The deformation of the assembly involves creation and loss of contacts as well as restructuring by means of rolling and sliding contacts. These changes imply a continuous variation of the stress-strain relation and a change of the void ratio during load. The stress tensor is calculated from the forces applied on the boundary of the sample as $\sigma_{ij} = \frac{1}{A}\sum_b f_i^b x_j^b$, where $\vec{x}^b$ is the point of application of the boundary force $\vec{f}^b$ and A is the area enclosed by the membrane. From the principal values of this tensor, one can define the mean normal stress $p = (\sigma_1 + \sigma_2)/2$ and the deviatoric stress $q = (\sigma_1 - \sigma_2)/2$. The axial strain is calculated as $\epsilon_1 = \Delta H/H_0$, where H is the height of the sample. The volumetric strain is given by $\epsilon_V = \Delta A/A_0$.

The dependence of the deviatoric stress and the volumetric strain on the axial strain are shown in Fig. 1 for different confining pressures. A continuous decrease of the initial slope of the stress-strain curve

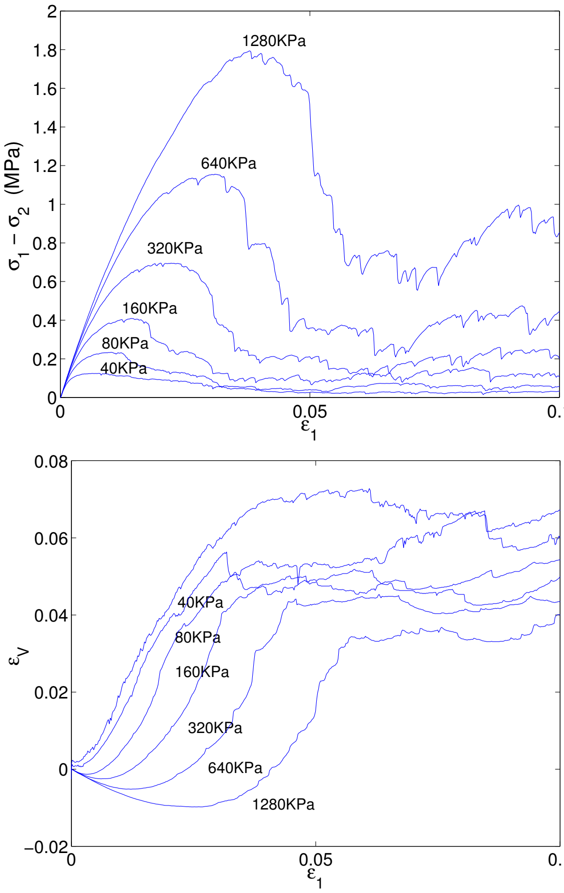

Figure 1: Deviatoric stress and volumetric strain versus axial strain for different confining pressures.

is observed. Loading rearranges the contact network by means of sliding contacts, which in turn reduces the stiffness of the material. The initial compaction turns gradually to dilatancy. This transition is caused by loss of contacts perpendicular to the load direction, allowing the contact network to rearrange itself and inducing large plastic deformations. Near failure, the amount of plastic deformation is much larger than the elastic one. This considerably reduces the stiffness with respect to its initial value and makes the sample potentially unstable.

Plastic deformation by means of sliding contacts turns out to be a precursor mechanism of strain localizations. The frictional dissipation is uniformly distributed at the beginning of the load and it tends progressively to localize in thin layers, which ends up with the shear band formation (Alonso-Marroquin 2004). Near failure, the orientational distribution of sliding contacts has its maximal value between the Mohr-Coulomb angle and the Roscoe angle, but rather closer to the former (Alonso-Marroquin, Luding, Herrmann, and Vardoulakis 2004). The shear band is given by a $6-8$ grain diameters thick layer where the frictional dissipation is more intense than on the average.

The characteristic width of the shear of the band

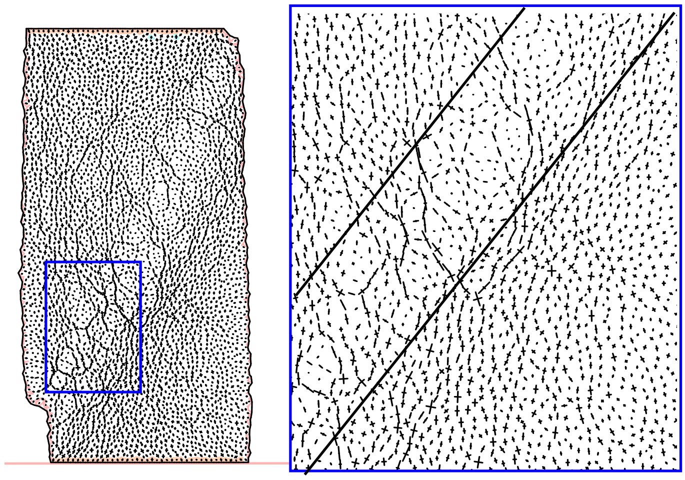

Figure 2: Principal stress directions of the individual grains after failure ($\epsilon_1 = 0.07$). The confining pressure is $p_0 = 160KPa$.

can be associated to the propagation of stress inside the grains. The stress tensor at each particle P is given by $\sigma_{ij}^P = \frac{1}{a}\sum_c f_i^c \ell_j^c$ where a is the area of the polygon, f_i^c is the contact force and ℓ_j^c is the branch vector, connecting the center of mass of the polygon to the point of application of the contact force. The sum goes over all the contacts of the particle. The principal stress direction at each grain is represented in Fig. 2 by a cross. The length of the lines represents how large the components are. At the beginning of the loading, the major principal stress is almost parallel to the load direction, forming column-like structures which are usually called chain forces. At failure these chain forces start buckling, and the buckled chains gradually concentrate as shear bands. The shear band width corresponds to the characteristic length of such buckles, that does not depend on the sample size. Bucking of each chain force involves rolling and sliding between the grains belonging to it, a feature which has been used to provide a theoretical explanation of the finite width of shear bands (Satake 1998).

3 FLUCTUATION INSIDE SHEAR BANDS

The Critical State Soil Mechanics presumes that for large shear deformations a soil element will reach asymptotically a limiting state characterized by an isochoric deformation, where the stress ratio and the frictional dissipation stay constant (Wood 1990). Numerical simulations using polygonal packings show that samples with different densities reach the same critical state, where the density and the stress ratio stay approximately constant, except for some fluctuations (Peña, Lizcano, Alonso-Marroquin, and Herrmann 2004). These fluctuations are reflected in the probability distribution of grain displacements, which follows approximately a power law as is the case in tectonic faults (Tillemans and Herrmann 1995). Sim-

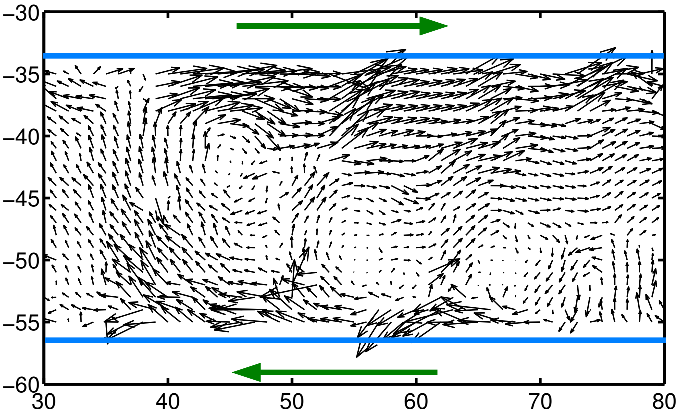

Figure 3: Displacement field in the shear cell

ulation shows also that different contact friction coefficients lead to the same critical state (Peña, Lizcano, Alonso-Marroquin, and Herrmann 2004). Therefore this state does not depend only on interparticle friction, but further kinematical modes such as rolling should affect this state. Recently it has been indicated that due to collapsing of microstructure, the critical state is approached on the average in the sense of dilatancy and compaction fluctuations (Vardoulakis and Georgopoulos 2004).

These fluctuations become apparent in simulations of periodic shear cells. The cells consist of disks with diameters between $0.4cm$ and $1.2cm$. The normal and tangential stiffnesses at the contact are $k_n = 160MPa$ and $k_t = 16MPa$. The friction coefficient is $\mu = 0.5$. We chose periodical boundary conditions in the horizontal direction. A normal stress of $\sigma = 16kPa$ and a constant horizontal velocity of $v_x = 0.5mm/s$ in opposite directions are imposed on the confining plates, see Fig. 3. The length of the cell is $120cm$ and the thickness of the disks is $1cm$.

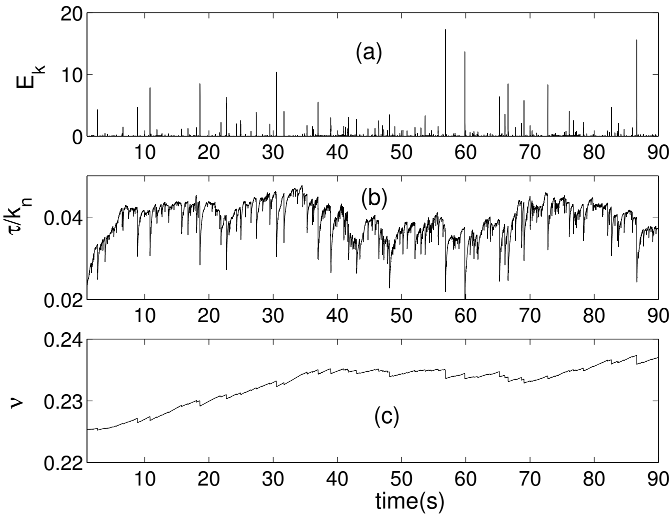

Figure 4: (a) Mean kinetic energy, (b) shear stress and (c) void ratio versus time.

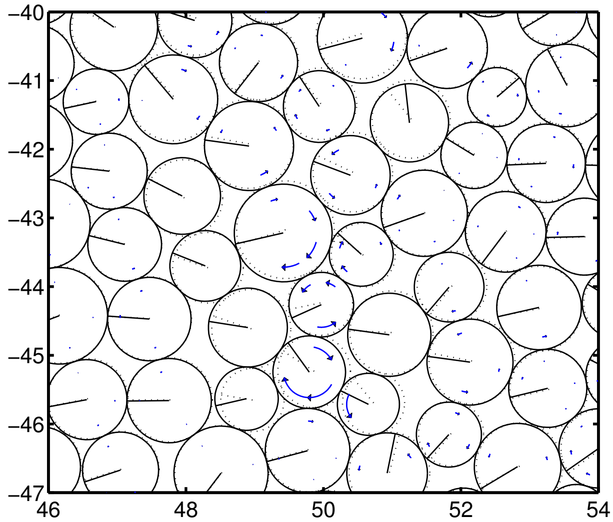

Figure 5: Displacement at the contacts.

The dynamics of the shear cell resembles in many aspects the slip-stick activity in tectonic plates. This is shown in Fig. 4, where we plot (a) the mean kinetic energy, (b) the shear force $\tau = (F_x^{top} - F_x^{bot})/2L$ at the walls (F_x are the horizontal component of the forces applied on each plate) and (c) void ratio. The energy stored during the shear is released in the form of *quakes*. Each quake corresponds to a collapse of stress columns which is reflected on an abrupt drop of stress and void ratio. Between two quakes the stress columns build up and the frictional contacts increase leading to an elastoplastic response with constant dilatancy.

Fig. 3 demonstrates that granular media do not support the idea of simple shear. The observed kinematic suggests that at any time we deal with two populations of grains: (a) Grains organized in large vorticity cells and (b) grains which through pronounced rolling accommodate the cells to make them more compatible with the imposed kinematic boundary conditions. Rolling at one contact can be visualized by marking two points at each particle near to the contact. After a small time interval, these points as well as the contact point, are translated (Bagi and Kuhn 2004). The translation of the contact at each grain is given by a vector connecting the current position of the contact with the marked point (Alonso-Marroquin, Froiio, and Vardoulakis 2005). These vectors are plotted in Fig. 5. We observe localized zones between the vorticity cells where rolling is more frequent than sliding.

The main effect of rolling is to reduce the frictional dissipation with respect to simple shear. The theoretical power dissipation of simple shear is given by $W = \mu\sigma L v d$ where μ is the friction coefficient; σ the normal stress; L the length of the shear cell; v the relative velocity of the plates; and d is the thickness of the disks. The theoretical value is given by $W = 0.096\ watts$. This energy is bigger than the computed mean frictional dissipation in the packing, which is $W \approx 0.06\ watts$ (Alonso-Marroquin, Froiio, and Vardoulakis 2005). (Viscous dissipation is negligible). This reduction of dissipation due to rolling can be associated to the lack of heat flux in seismic gaps, where rolling between the rocks may play also an important role (Mora and Place 1999).

4 CONCLUSIONS

We have numerically studied the role of sliding and rolling in shear bands. We observe a progressive localization of sliding contacts starting from the beginning of the loading. Buckling of stress columns ends up with shear bands with a characteristic width of $6 - 8$ grain diameters. Building of stress columns with constant dilatancy and collapse of columns in the form of quakes characterizes the dynamics of the shear band at the so-called critical state. Vorticity cells assisted by rolling lead to a significant reduction of heat dissipation with respect to the value given for simple shear. The question that naturally arises is: Can these rotational models explain the lack of seismic activity and heat dissipation measures in the seismic gaps?

In realistic shear bands, grains tend to fragment leading to dense packings of grains of very different sizes. Do rotational bearings exist in such dense materials?. In the case of spherical grains, it is possible to construct space filling configurations iteratively (Mahmoodi-Baran, Herrmann, and Rivier 2004). Some of these configurations have a surprising property: If one imposes rotation to any single sphere, all others will rotate without any slip at the contacts. Although in reality there are no such perfect bearing packings, the rotational modes we observe suggest the possibility of spontaneous formation of vorticity cells, where rolling dominates over sliding. As the continuum theory is concerned, the Cosserat theory would be essential to capture the effect of rolling in the overall deformation (Vardoulakis and Sulem 1995; Mühlhaus and Vardoulakis 1987). Apart from the stress, the couple stress tensor should be introduced as a static variable, entering as the counterpart of the rotational strain in the power dissipation (Tordesillas, Walsh, and Gardiner 2004; Ehlers, Ramm, Diebels, and D´Addetta 2003). A better understanding of shear bands and earthquakes may require a multi-scale analysis of the Cosserat theory, connecting the micromechanical tractions and deformations at individual contacts to their global effects at the large scale of the seismic gaps.

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